Catastrophe Reinsurance Risk – A Unique Asset Class

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Outline

- The natural catastrophe reinsurance market
- Characteristics of natural catastrophes and risk quantification
- Opportunities for investors – reinsurance risk as an asset class
- Reinsurance risk selection and portfolio construction
The natural catastrophe reinsurance market

• Catastrophe risk is a major challenge to the property insurance industry
  – Significant capital requirement limits return on equity
  – Earning volatilities suppress stock valuation
  – Natural perils are the main sources of catastrophe risk

• Catastrophe reinsurance is the most effective tool for insurers to manage the risk
  – Customized to fit insurer’s risk management needs
  – Adjusted periodically to reflect the insurer’s evolving risk profile
Forms of catastrophe reinsurance

• Traditional reinsurance
  – Sold by reinsurance companies
  – To the buyer of reinsurance
    Pros: highly customized indemnity cover, long-term business partnership
    Cons: limited capacity for cat risk, can be expensive, counter-party risk

• Capital market solutions
  – Direct participation in risk taking by investors: cat bonds, etc.
  – To the buyer of reinsurance
    Pros: increased market capacity, pricing stability (many are multi-year contracts), virtually no counter-party risk
    Cons: basis risk due to the lack of indemnity cover (mostly index or parametric), high structuring cost for small insurers or reinsurers
Characteristics of natural catastrophe risk

• Low frequency but high severity
  – Lack of historical data
  – Uncertainty in scientific understanding of extreme events
  – Impact of global climate change
  – Seismic stress buildup
  – Building performance changes over time

• Extreme concentration of property value
  – Impractical to manage risk by diversification
  – High risk premiums for peak exposed areas
  – Premium can be a very large multiple of expected loss
Quantification of natural catastrophe risk

• Limitation of traditional approaches for the purpose of pricing natural catastrophe risk transfers
  – No-arbitrage option pricing model – underlying risk not traded
  – Traditional statistical methods – not enough data

• Catastrophe models
  – Built on science and engineering studies of natural hazards
  – Simulation-based
  – Emerged ~20 years ago and gaining acceptance over the years by insurers, reinsurers, and rating agencies
Catastrophe models

Math concept behind cat models

- $X$ = hazard measure (e.g., wind speed);
- $Y$ = property damage
- $Z$ = insurance loss

Cat events $\Rightarrow f_X(x)$

Vulnerability $\Rightarrow f_{Y|X}(y \mid X = x)$

$f_Y(y) = \int f_{Y|X}(y \mid x) f_X(x) dx$

Insurance contract terms $\Rightarrow Z = Z(Y)$
Reinsurance risk as an asset class

- Insurance-linked securities (ILS) are financial instruments whose performance are primarily driven by insurance and/or reinsurance loss events
  - Narrowly defined: 144A securities whose coupon and interest payments are determined by the frequency and severity of insurance or reinsurance loss events. These are known as *cat bonds*
  - Broadly, include cat bonds + private insurance and reinsurance transactions in various forms
  - In the broadest sense: all above + stocks and bonds of insurance and reinsurance companies
  - We focus on the ones linked to natural catastrophes
Reinsurance risk as an asset class

Source: Swiss Re, Barclays Capital
Reinsurance risk as an asset class

• Many ILS linked to natural catastrophes offer attractive risk-adjusted returns

• They are also generally uncorrelated with the overall financial market performance: notable exceptions
  – Extremely large catastrophe event
  – ILS valuation still subject to liquidity risk

• All factors considered, ILS is an attractive asset class
Risk selection and portfolio construction

- An reinsurance risk portfolio = a collection of reinsurance contracts (cat bonds and private transactions) + hedges
- An optimal portfolio
  - Maximizes return at a given acceptable level of risk
  - Minimizes risk at a required rate of return

![Graph showing the efficient frontier and optimal portfolios.](image)
Risk and return measures

- **Return**
  - Expected profit = premium - expected loss – expenses

- **Risk:**
  - Standard deviation
  - Occurrence probable maximum loss (PML)
  - Value at Risk (VaR)
  - Tail Value at Risk (TVaR)
  - Maximum possible loss (MPL)

- No single “best” choice
Risk selection and portfolio construction

• Goal: From the universe of eligible instruments, select risk-taking and hedging positions to construct a portfolio that (a) conforms to a set of risk constraints and (b) maximizes the expected profit.
Risk selection and portfolio construction

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• Keys to success:
  • Access to the complete universe of instruments
  • Ability to analyze the instruments and their interdependence
Theoretical framework

- Expected portfolio profit

\[
E(P) = \sum_{i=1}^{n} w_i \pi_i - \sum_{i=1}^{n} w_i E(L_i)
\]

- \( E(\ ) \) = expected value operator
- \( n \) = number of instruments in the “universe” of transactions
- \( P \) = profit of the portfolio
- \( L_i \) = loss of the \( i \)th contract
- \( \pi_i \) = premium of the \( i \)th contract (net of all expenses)
- \( w_i \) = position (i.e. amount of risk taken) of the \( i \)th contract

Take risk (e.g., selling insurance, reinsurance, buying cat bonds): \( w_i > 0 \)

Hedge risk (e.g., buying reinsurance, issuing cat bonds): \( w_i < 0 \)
Theoretical framework

• Constraints:
  - Key risk and return measures are bound by specific thresholds

\[
c_k^l \leq c_k \left( \sum_{i=1}^{n} w_i \pi_i, \sum_{i=1}^{n} w_i L_i \right) \leq c_k^u
\]

\[k = 1, 2, \ldots, m\]

\[c_k = \text{the } k^{\text{th}} \text{ constraint function}\]

\[c_k^l \text{ and } c_k^u = \text{lower and upper bounds of the } k^{\text{th}} \text{ constraint}\]

  - Realistic range of risk position \(w_i\)

\[
w_i^l \leq w_i \leq w_i^u
\]

\[w_i^l \text{ and } w_i^u = \text{lower and upper bounds of } w_i\]
Theoretical framework

- Given a set of $w_i$ values, the return and risk of the portfolio can be calculated, i.e., each portfolio (a dot in the chart above) is determined by a set of $w_i$ values.

- Hence, we are looking for the sets of $w_i$ that put the portfolio on (or at least close to) the efficient frontier, i.e., $w_i = \text{our solution space}$.
Mathematical / numerical solutions

• Linear programming works for special cases
  – When constraints and risk functions are linear with respect to contract positions
  – When TVaR is used as the constraint and the objective is to maximize expected profit, the problem can be converted to a linear programming problem

• In practice, few traditional approaches work
  – Risk constraints are not linear or smooth, creating many local suboptimal solutions
  – Dimension is too high for exhaustive search or other numerically demanding search algorithms
Mathematical / numerical solutions

• A working approach must be
  – computationally efficient and scalable
  – Able to handle non-linear non-smooth risk functions
  – Robust with respect to parameter uncertainty
  – Produce substantially better results than benchmarks (see next slide)

• Two working approaches
  – Genetic algorithm
  – Simulated annealing

  – It is beyond the scope of this presentation to address the details of these approaches
Real-world application: example

- Investment decisions by the manager of a cat bond fund
  - Goal: determine the optimal amount of cat bonds to purchase for the fund
  - Objective: maximize return
  - Risk Constraints for the fund
    - Portfolio 100-year VaR < $55M
    - Maximum holding of each cat bond < 5M
  - Market access constraints
    - Lower bound =0 (impractical to short cat bonds)
    - There is an upper limit on the amount that the manager can possibly buy because the market is illiquid
    - The manager has a finite amount of capital to deploy

- Question: How to determine how much to invest in each cat bond available in the market
Real-world application: example

• Use portfolio optimization to accomplish this task: concrete steps
  – Model the universe of bonds
    • Model each bond to create simulated losses by event for each bond, where the simulated loss are every unit of capital invested
  – Define the solution space (i.e., what to optimize)
    • Amount to invest in each bond
  – Establish the upper bounds of the solution space
    • Maximum allowed to invest in each bond
  – Estimate the price of each cat bond
    • Needed to calculated expected return
  – Specify the risk constraints in the optimization tool
  – Run the optimization tool ➔ obtain the solution: the amount to invest in each cat bond
Real-world application: example

• To demonstrate the value of optimization, construct three portfolios using two benchmark methods in addition to optimization
  – Equal amount invested in each bond (Benchmark 1)
  – Portfolio selection based on ranking of individual risk/return characteristics, (Benchmark 2)
    • Rank the cat bonds in the universe by their individual risk/return
    • Make the maximum possible investment in each bond in the order above until the total capital is completely deployed or the risk constraint is reached

• Compare the risk/return profiles of the portfolios constructed using these three methods
## Real-world application: example

<table>
<thead>
<tr>
<th>Improvement of profitability given the same risk constraint</th>
<th>expected profit</th>
<th>100-year TVaR</th>
<th>Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2) over (1)</td>
<td>11%</td>
<td>17%</td>
<td></td>
</tr>
<tr>
<td>(3) over (2)</td>
<td>31%</td>
<td>16%</td>
<td></td>
</tr>
<tr>
<td>(3) over (1)</td>
<td>46%</td>
<td>36%</td>
<td></td>
</tr>
</tbody>
</table>

| (1) equal investment in each bond                           | 7,247,880       | 55,000,000     | 66%          |
| (2) rank by individual risk/return                          | 8,066,907       | 55,000,000     | 77%          |
| (3) optimized                                               | 10,586,927      | 55,000,000     | 90%          |
Summary

- Financial instruments linked to natural catastrophe reinsurance risks represent a unique asset class since they offer attractive risk-adjusted returns that are generally uncorrelated with the overall financial market.

- The unique characteristics of natural catastrophes and the cat risk market present a challenge to reinsurance companies and investors in such risks.

- They also present “Alpha” opportunities to diligent investors with substantial investment in analytics.