Mortgage OAS Analysis

Harvey Stein

Head, Quantitative Finance R&D
Bloomberg LP

16th February 2006

OAS calculations, Monte Carlo, OAS server and Linux cluster work by Alexander Belikoff, Kirill Levin, Harvey Stein, and Xusheng Tian, Quantitative Finance R&D group, Bloomberg LP.
Prepayment modeling by Warren Xia and Sherman Liu, Prepayment Modeling Group, Bloomberg LP.
OAS application by Sean Dai, Mortgage group, Bloomberg LP.
Thanks to Bruno Dupire and Liuren Wu, Quantitative Finance R&D group, Bloomberg LP, for assistance.
Outline

1. Customary market size comments
2. Mortgage market structure
3. Prepayment modeling
4. Yield and OAS
5. Data and calibration
6. Interest rate models
7. Index projection
8. Monte Carlo analysis
9. Greeks
10. Validation
11. Robust parallelization
12. Summary
Size of mortgage market

- The Case for U.S. Mortgage-Backed Securities for Global Investors, Michael Wands, CFA, Head of U.S. Fixed Income, Global Fixed Income, State Street Global Advisors:
  - Lehman U.S. Aggregate Index - 2002
    - MBS — 35%
    - U.S. Credit — 27%
    - U.S. Treasury — 22%
    - U.S. Agency — 12%
    - ABS — 2%
    - CMBS — 2%

- The U.S. Mortgage Market, Fannie Mae, and Freddie Mac — An IMF Study:
  - March 2003 — $3.2 trillion in mortgage-backed issuance by Fannie and Freddie.
## Outstanding debt

<table>
<thead>
<tr>
<th>Source</th>
<th>Federal Reserve</th>
<th>Current Value</th>
<th>Date</th>
<th>Previous Value</th>
<th>Date</th>
<th>Pct Chng</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Debt Outstanding Total</td>
<td>DOUTTOTL</td>
<td>25742.1</td>
<td>09/05</td>
<td>25168.0</td>
<td>06/05</td>
<td>2.28</td>
</tr>
<tr>
<td>2) Debt Outstanding Fed Govt</td>
<td>DOUTFED</td>
<td>4612.1</td>
<td>09/05</td>
<td>4554.1</td>
<td>06/05</td>
<td>1.27</td>
</tr>
<tr>
<td>3) Debt Outstanding Nonfed Tot</td>
<td>DOUTNONT</td>
<td>21130.0</td>
<td>09/05</td>
<td>20613.9</td>
<td>06/05</td>
<td>2.50</td>
</tr>
<tr>
<td>4) Debt Outstanding Houshld</td>
<td>DOUTHHL</td>
<td>11000.3</td>
<td>09/05</td>
<td>10691.3</td>
<td>06/05</td>
<td>2.89</td>
</tr>
<tr>
<td>5) Debt Outstanding Home Mrtge</td>
<td>DOUTMORT</td>
<td>8208.9</td>
<td>09/05</td>
<td>7932.3</td>
<td>06/05</td>
<td>3.49</td>
</tr>
<tr>
<td>6) Debt Outstanding Cnsmr Crdt</td>
<td>DOUTCON</td>
<td>2193.5</td>
<td>09/05</td>
<td>2164.3</td>
<td>06/05</td>
<td>1.35</td>
</tr>
<tr>
<td>7) Debt Outstanding Business</td>
<td>DOUTBUS</td>
<td>8319.6</td>
<td>09/05</td>
<td>8167.6</td>
<td>06/05</td>
<td>1.86</td>
</tr>
<tr>
<td>8) Debt Outstanding Corporate</td>
<td>DOUTCOR</td>
<td>5486.3</td>
<td>09/05</td>
<td>5395.8</td>
<td>06/05</td>
<td>1.68</td>
</tr>
<tr>
<td>9) Debt Outstanding State&amp;Locl</td>
<td>DOUTSTAL</td>
<td>1810.1</td>
<td>09/05</td>
<td>1754.9</td>
<td>06/05</td>
<td>3.15</td>
</tr>
<tr>
<td>10) Debt Outstanding Domstc Fin</td>
<td>DOUTDOM</td>
<td>12260.9</td>
<td>09/05</td>
<td>12089.9</td>
<td>06/05</td>
<td>1.41</td>
</tr>
<tr>
<td>11) Debt Outstanding Foreign</td>
<td>DOUTFRGN</td>
<td>945.3</td>
<td>09/05</td>
<td>917.4</td>
<td>06/05</td>
<td>3.04</td>
</tr>
</tbody>
</table>
Size over time
MBS issuance

<table>
<thead>
<tr>
<th>Issuer</th>
<th>Jan</th>
<th>Dec</th>
<th>Nov</th>
<th>Oct</th>
<th>Sep</th>
<th>Aug</th>
<th>Jul</th>
<th>Jun</th>
<th>May</th>
<th>Apr</th>
<th>Mar</th>
</tr>
</thead>
<tbody>
<tr>
<td>FHLMC</td>
<td>35.6</td>
<td>34.2</td>
<td>37.7</td>
<td>34.7</td>
<td>42.4</td>
<td>36.9</td>
<td>28.6</td>
<td>29.7</td>
<td>29.6</td>
<td>32.0</td>
<td>25.9</td>
</tr>
<tr>
<td>FNMA</td>
<td>45.5</td>
<td>42.1</td>
<td>42.7</td>
<td>47.4</td>
<td>65.2</td>
<td>50.3</td>
<td>49.2</td>
<td>41.8</td>
<td>37.7</td>
<td>41.9</td>
<td>33.5</td>
</tr>
<tr>
<td>GNMA1</td>
<td>2.7</td>
<td>2.9</td>
<td>3.3</td>
<td>3.7</td>
<td>4.4</td>
<td>3.8</td>
<td>4.1</td>
<td>3.4</td>
<td>3.7</td>
<td>3.6</td>
<td>3.2</td>
</tr>
<tr>
<td>GNMA2</td>
<td>3.6</td>
<td>2.5</td>
<td>3.1</td>
<td>3.7</td>
<td>3.8</td>
<td>3.4</td>
<td>4.3</td>
<td>3.6</td>
<td>3.9</td>
<td>3.8</td>
<td>3.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Loan Type</th>
<th>Jan</th>
<th>Dec</th>
<th>Nov</th>
<th>Oct</th>
<th>Sep</th>
<th>Aug</th>
<th>Jul</th>
<th>Jun</th>
<th>May</th>
<th>Apr</th>
<th>Mar</th>
</tr>
</thead>
<tbody>
<tr>
<td>30yr</td>
<td>71.9</td>
<td>65.5</td>
<td>73.0</td>
<td>72.6</td>
<td>84.9</td>
<td>74.4</td>
<td>65.8</td>
<td>57.5</td>
<td>54.3</td>
<td>56.8</td>
<td>44.9</td>
</tr>
<tr>
<td>15yr</td>
<td>5.6</td>
<td>6.0</td>
<td>3.9</td>
<td>4.6</td>
<td>6.2</td>
<td>6.8</td>
<td>5.6</td>
<td>5.4</td>
<td>6.5</td>
<td>7.3</td>
<td>7.0</td>
</tr>
<tr>
<td>ARM</td>
<td>7.9</td>
<td>7.3</td>
<td>6.8</td>
<td>9.1</td>
<td>21.0</td>
<td>9.8</td>
<td>11.5</td>
<td>12.7</td>
<td>11.2</td>
<td>13.5</td>
<td>11.0</td>
</tr>
<tr>
<td>Other</td>
<td>1.9</td>
<td>3.0</td>
<td>3.1</td>
<td>3.1</td>
<td>3.7</td>
<td>3.2</td>
<td>3.3</td>
<td>2.9</td>
<td>2.9</td>
<td>3.7</td>
<td>3.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coupon (30yr Fixed)</th>
<th>Jan</th>
<th>Dec</th>
<th>Nov</th>
<th>Oct</th>
<th>Sep</th>
<th>Aug</th>
<th>Jul</th>
<th>Jun</th>
<th>May</th>
<th>Apr</th>
<th>Mar</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;4.5-</td>
<td>3.9</td>
<td>3.1</td>
<td>4.6</td>
<td>3.5</td>
<td>2.3</td>
<td>3.2</td>
<td>4.5</td>
<td>2.2</td>
<td>2.1</td>
<td>4.1</td>
<td>2.3</td>
</tr>
<tr>
<td>4.5-</td>
<td>1.6</td>
<td>1.8</td>
<td>3.0</td>
<td>6.5</td>
<td>13.0</td>
<td>8.6</td>
<td>4.2</td>
<td>2.9</td>
<td>3.1</td>
<td>3.2</td>
<td>3.5</td>
</tr>
<tr>
<td>5.0-</td>
<td>15.3</td>
<td>20.5</td>
<td>25.7</td>
<td>26.5</td>
<td>39.3</td>
<td>35.3</td>
<td>33.5</td>
<td>25.9</td>
<td>21.0</td>
<td>23.8</td>
<td>14.2</td>
</tr>
<tr>
<td>5.5-</td>
<td>26.0</td>
<td>25.0</td>
<td>29.0</td>
<td>29.2</td>
<td>24.3</td>
<td>21.4</td>
<td>18.0</td>
<td>19.3</td>
<td>21.4</td>
<td>20.8</td>
<td>20.7</td>
</tr>
<tr>
<td>6.0-</td>
<td>20.5</td>
<td>12.3</td>
<td>8.7</td>
<td>5.8</td>
<td>4.9</td>
<td>5.0</td>
<td>4.8</td>
<td>6.0</td>
<td>5.7</td>
<td>4.1</td>
<td>3.1</td>
</tr>
<tr>
<td>6.5-</td>
<td>3.7</td>
<td>2.1</td>
<td>1.4</td>
<td>.8</td>
<td>.7</td>
<td>.5</td>
<td>.5</td>
<td>.7</td>
<td>.7</td>
<td>.6</td>
<td>.8</td>
</tr>
<tr>
<td>7.0-</td>
<td>.9</td>
<td>.7</td>
<td>.5</td>
<td>.3</td>
<td>.3</td>
<td>.3</td>
<td>.2</td>
<td>.3</td>
<td>.2</td>
<td>.2</td>
<td>.2</td>
</tr>
<tr>
<td>&gt;7.5</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
<td>.2</td>
</tr>
</tbody>
</table>

Summary

Mortgage OAS Analysis
Harvey Stein

Customary market size comments
Mortgage market structure
Prepayment modeling
Yield and OAS
Data and calibration
Interest rate models
Index projection
Monte Carlo analysis
Greeks
Validation
Robust parallelization

Page 6 of 106
## CMO issuance

<table>
<thead>
<tr>
<th></th>
<th>Mtge</th>
<th>ICMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>165</td>
<td>165</td>
</tr>
<tr>
<td>Feb</td>
<td>165</td>
<td>165</td>
</tr>
<tr>
<td>Mar</td>
<td>165</td>
<td>165</td>
</tr>
<tr>
<td>Apr</td>
<td>165</td>
<td>165</td>
</tr>
<tr>
<td>May</td>
<td>165</td>
<td>165</td>
</tr>
<tr>
<td>Jun</td>
<td>165</td>
<td>165</td>
</tr>
<tr>
<td>Jul</td>
<td>165</td>
<td>165</td>
</tr>
<tr>
<td>Aug</td>
<td>165</td>
<td>165</td>
</tr>
<tr>
<td>Sep</td>
<td>165</td>
<td>165</td>
</tr>
<tr>
<td>Oct</td>
<td>165</td>
<td>165</td>
</tr>
<tr>
<td>Nov</td>
<td>165</td>
<td>165</td>
</tr>
<tr>
<td>Dec</td>
<td>165</td>
<td>165</td>
</tr>
</tbody>
</table>

### Collateral Source

<table>
<thead>
<tr>
<th>Source</th>
<th>Mtge</th>
<th>ICMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>FHLMC</td>
<td>15.2</td>
<td>15.2</td>
</tr>
<tr>
<td>FNMA</td>
<td>10.7</td>
<td>10.7</td>
</tr>
<tr>
<td>GNMA</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>WHOLE</td>
<td>12.8</td>
<td>13.0</td>
</tr>
</tbody>
</table>

### Collateral Type

<table>
<thead>
<tr>
<th>Type</th>
<th>Mtge</th>
<th>ICMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>30yr</td>
<td>23.0</td>
<td>23.0</td>
</tr>
<tr>
<td>15yr</td>
<td>14.4</td>
<td>14.4</td>
</tr>
<tr>
<td>ARM</td>
<td>15.2</td>
<td>15.2</td>
</tr>
<tr>
<td>Other</td>
<td>7.1</td>
<td>8.0</td>
</tr>
</tbody>
</table>

### Collateral WAC (Fixed Rate)

<table>
<thead>
<tr>
<th>Rate</th>
<th>Mtge</th>
<th>ICMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;5.5</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>5.5-</td>
<td>7.6</td>
<td>14.4</td>
</tr>
<tr>
<td>6.0-</td>
<td>14.4</td>
<td>28.9</td>
</tr>
<tr>
<td>6.5-</td>
<td>1.0</td>
<td>4.0</td>
</tr>
<tr>
<td>7.0-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7.5-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8.0-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>&gt;8.5</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### Summary

- CMO issuance for 2004-2006
- Data and calibration
- Interest rate models
- Index projection
- Monte Carlo analysis
- Greeks
- Validation
- Robust parallelization
- Summary
Mortgage market structure:

- Mortgages
- MBS pools
- CMOs
The subatomic particles - mortgages

People take out loans (mortgages) to buy homes.

- Fixed rate mortgages — fixed coupon, monthly payments, self amortizing, paying principal down to zero at maturity (15-30 years).
- Balloons – amortize on a 30 year basis, but expire in 5 or 7 years with payment of the remaining outstanding balance.
- Adjustable rate mortgages (ARMS) — floating coupon based on an index (LIBOR, treasury rates, ...), typically with protection clauses against overly large coupon changes (lifetime and periodic caps, annual resets, ...).

Low interest rates in recent years have sparked innovation — floating rate balloons with interest only payments, various sorts of built in protections, option ARMs, ...
Mortgage behavior

While it looks like ARM valuation might require some work, in that they have embedded caps and ratchets and potentially float on a CMS rate, one might think that at least fixed rate mortgages would be easily valued.

With fixed monthly payments, monthly interest payments at a rate of $C$ on the outstanding balance, $N$ monthly payments and an initial balance of $B$, then monthly payments are:

$$\frac{C(1+C)^NB}{(1+C)^N - 1}.$$
Mortgage cash flows

Graphically, our cash flows look like:

so why not just discount and be done?
The painful intervention of reality

The problem with discounting the scheduled payments is that

- People move,
- refinance,
- default,
- make excess payments to pay off principal faster.

So, principal arrives randomly, or perhaps not at all (in default of uninsured loans).

Payments above the scheduled payments pay down the principal and are known as *prepayment*.

One of the major components of mortgage analysis is in modeling prepayment behavior.
Impact of prepayment on cash flows

Thus, our simple mortgage, instead of having fixed cash flows, has cash flows that are dependent on prepayment rates. Under one prepayment assumption, we have:
Impact of prepayment on cash flows

Under a model that projects prepayments based on interest rates and loan characteristics, we see a different cash flow structure for each rate path. One example is:
In 1938, the collapse of the national housing market led to the federal government’s formation of the *Federal National Mortgage Association*, AKA “Fannie Mae”. Now, we have Fannie Mae, Freddie Mac (*The Federal Home Loan Mortgage Corporation*), and Ginnie Mae *The Government National Mortgage Association*.

- Banks make mortgages.
- Government insures *conforming* mortgages.
- Agencies buy conforming mortgages.
- Banks have money to make more mortgages.
- Agencies sell shares of mortgages on secondary market.

Ginnie, Fannie and Freddie — three sets of rules for conforming loans.
MBS secondary market - pools

The agencies (Fannie, Freddie and Ginnie) buy mortgages, pool them together into MBS pools and sell shares. Pools are pass-through securities, in that the cash flows from the underlying collateral is passed through to the shareholder, minus a service fee.
MBS pool fine characteristics - Geographic data

These days, there’s substantial information available about pool composition, such as geographic data:

<table>
<thead>
<tr>
<th>STATE</th>
<th>ORIG BALANCE</th>
<th>% OF TOTAL ORIG BALANCE</th>
<th>ORIG LOANS</th>
<th>% OF TOTAL ORIG LOANS</th>
<th>CURR BAL</th>
<th>% OF TOTAL CURR BAL</th>
<th>LOANS</th>
<th>% OF TOTAL LOANS</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL</td>
<td>192,538.02</td>
<td>1.68</td>
<td>2</td>
<td>2.90</td>
<td>1.58</td>
<td>2</td>
<td>2.90</td>
<td></td>
</tr>
<tr>
<td>AR</td>
<td>269,773.17</td>
<td>2.36</td>
<td>1</td>
<td>1.45</td>
<td>2.36</td>
<td>1</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>AZ</td>
<td>1,080,851.01</td>
<td>9.44</td>
<td>7</td>
<td>10.14</td>
<td>9.44</td>
<td>7</td>
<td>10.14</td>
<td></td>
</tr>
<tr>
<td>CA</td>
<td>600,824.98</td>
<td>7.00</td>
<td>4</td>
<td>5.80</td>
<td>7.00</td>
<td>4</td>
<td>5.80</td>
<td></td>
</tr>
<tr>
<td>CT</td>
<td>506,204.00</td>
<td>7.92</td>
<td>4</td>
<td>5.80</td>
<td>7.92</td>
<td>4</td>
<td>5.80</td>
<td></td>
</tr>
<tr>
<td>FL</td>
<td>1,427,623.42</td>
<td>12.47</td>
<td>8</td>
<td>11.59</td>
<td>12.47</td>
<td>8</td>
<td>11.59</td>
<td></td>
</tr>
<tr>
<td>IA</td>
<td>81,533.11</td>
<td>0.71</td>
<td>1</td>
<td>1.45</td>
<td>0.71</td>
<td>1</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>ID</td>
<td>119,094.12</td>
<td>1.05</td>
<td>1</td>
<td>1.45</td>
<td>1.05</td>
<td>1</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>IL</td>
<td>712,887.56</td>
<td>6.23</td>
<td>5</td>
<td>7.25</td>
<td>6.23</td>
<td>5</td>
<td>7.25</td>
<td></td>
</tr>
<tr>
<td>IN</td>
<td>76,935.31</td>
<td>0.67</td>
<td>1</td>
<td>1.45</td>
<td>0.67</td>
<td>1</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>KS</td>
<td>131,889.10</td>
<td>1.15</td>
<td>1</td>
<td>1.45</td>
<td>1.15</td>
<td>1</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>MI</td>
<td>471,994.57</td>
<td>4.12</td>
<td>4</td>
<td>5.80</td>
<td>4.12</td>
<td>4</td>
<td>5.80</td>
<td></td>
</tr>
<tr>
<td>MN</td>
<td>186,147.29</td>
<td>1.63</td>
<td>1</td>
<td>1.45</td>
<td>1.63</td>
<td>1</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>NC</td>
<td>175,452.47</td>
<td>1.53</td>
<td>1</td>
<td>1.45</td>
<td>1.53</td>
<td>1</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>ND</td>
<td>50,355.53</td>
<td>0.44</td>
<td>1</td>
<td>1.45</td>
<td>0.44</td>
<td>1</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>NE</td>
<td>71,080.03</td>
<td>0.62</td>
<td>1</td>
<td>1.45</td>
<td>0.62</td>
<td>1</td>
<td>1.45</td>
<td></td>
</tr>
</tbody>
</table>
### Collateral Composition

**FG A41492**

As of Feb 2005

<table>
<thead>
<tr>
<th>Loan Purpose</th>
<th>#Loans</th>
<th>%Bal</th>
<th>$Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase</td>
<td>12</td>
<td>15.63</td>
<td>1,787,721</td>
</tr>
<tr>
<td>Refinance</td>
<td>57</td>
<td>84.37</td>
<td>9,650,036</td>
</tr>
<tr>
<td>Non-Reported</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Occupancy</th>
<th>#Loans</th>
<th>%Bal</th>
<th>$Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Owner Occup.</td>
<td>66</td>
<td>96.57</td>
<td>11,091,193</td>
</tr>
<tr>
<td>Vacation</td>
<td>3</td>
<td>3.03</td>
<td>346,564</td>
</tr>
<tr>
<td>Investor</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Non-Reported</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mortgaged Properties</th>
<th>#Loans</th>
<th>%Bal</th>
<th>$Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Family</td>
<td>67</td>
<td>97.43</td>
<td>11,143,807</td>
</tr>
<tr>
<td>2-4 Family</td>
<td>2</td>
<td>2.57</td>
<td>293,950</td>
</tr>
<tr>
<td>Non-Reported</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Servicer Breakdown Information Now On Page 9
MBS pool fine characteristics - Loan to value ratio and credit ratings

<table>
<thead>
<tr>
<th>Note Rate (%)</th>
<th>Minimum</th>
<th>25%</th>
<th>Median</th>
<th>75%</th>
<th>Maximum</th>
<th>Wgt'd. Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rem Mty (mths)</td>
<td>356</td>
<td>358</td>
<td>358</td>
<td>359</td>
<td>359</td>
<td>358</td>
</tr>
<tr>
<td>Loan Age (mths)</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Orig Term (mths)</td>
<td>360</td>
<td>360</td>
<td>360</td>
<td>350</td>
<td>350</td>
<td>360</td>
</tr>
<tr>
<td>DLS ($bal)</td>
<td>190,000</td>
<td>190,000</td>
<td>240,000</td>
<td>345,000</td>
<td>194,614*</td>
<td></td>
</tr>
<tr>
<td>DTV (%)</td>
<td>57</td>
<td>74</td>
<td>88</td>
<td>85</td>
<td>95</td>
<td>79</td>
</tr>
<tr>
<td>DCS</td>
<td>537</td>
<td>611</td>
<td>640</td>
<td>651</td>
<td>767</td>
<td>640</td>
</tr>
</tbody>
</table>

* Simple Average DLS = 166,043
MBS pool fine characteristics - loan rate distribution

Collateral Composition

FG A41492

Collateral Loan Rate Distribution as of Issuance

<table>
<thead>
<tr>
<th>LOAN RATE (PERCENT)</th>
<th>#</th>
<th>BAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.25-6.374</td>
<td>5</td>
<td>.9</td>
<td>7.5</td>
</tr>
<tr>
<td>6.625-6.749</td>
<td>6</td>
<td>.9</td>
<td>8.2</td>
</tr>
<tr>
<td>6.875-6.999</td>
<td>28</td>
<td>4.6</td>
<td>40.1</td>
</tr>
<tr>
<td>7.000</td>
<td>30</td>
<td>5.0</td>
<td>44.1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>69</td>
<td>11.4</td>
<td>100.0</td>
</tr>
</tbody>
</table>
### Amortized LTV Distribution as of Issuance

<table>
<thead>
<tr>
<th>LTV (%)</th>
<th>#</th>
<th>$MIL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>50-59</td>
<td>1</td>
<td>2</td>
<td>1.9</td>
</tr>
<tr>
<td>60-69</td>
<td>9</td>
<td>12</td>
<td>13.1</td>
</tr>
<tr>
<td>70-74</td>
<td>8</td>
<td>1.8</td>
<td>15.9</td>
</tr>
<tr>
<td>75-79</td>
<td>8</td>
<td>1.4</td>
<td>12.0</td>
</tr>
<tr>
<td>80-84</td>
<td>25</td>
<td>3.5</td>
<td>30.7</td>
</tr>
<tr>
<td>85-89</td>
<td>7</td>
<td>1.3</td>
<td>11.1</td>
</tr>
<tr>
<td>90-94</td>
<td>5</td>
<td>0.9</td>
<td>9.6</td>
</tr>
<tr>
<td>95-99</td>
<td>6</td>
<td>0.9</td>
<td>7.8</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>69</td>
<td>11.4</td>
<td>100.0</td>
</tr>
</tbody>
</table>
### MBS pool fine characteristics

#### Loan size distribution

<table>
<thead>
<tr>
<th>Loan Size ($000)</th>
<th>BAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-39</td>
<td>.3</td>
<td></td>
</tr>
<tr>
<td>50-59</td>
<td>.4</td>
<td></td>
</tr>
<tr>
<td>70-79</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>80-89</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>90-99</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>100-109</td>
<td>3.6</td>
<td></td>
</tr>
<tr>
<td>110-119</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>120-129</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>130-139</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td>140-149</td>
<td>8.7</td>
<td></td>
</tr>
<tr>
<td>150-159</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>160-169</td>
<td>7.1</td>
<td></td>
</tr>
<tr>
<td>170-179</td>
<td>9.2</td>
<td></td>
</tr>
<tr>
<td>180-189</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>190-199</td>
<td>5.1</td>
<td></td>
</tr>
<tr>
<td>200-209</td>
<td>5.3</td>
<td></td>
</tr>
</tbody>
</table>

| Total            | 11.4| 100.0 |

**Collateral Composition**

**FG A41492**

**Mortgage OAS**

**Analysis**

**Harvey Stein**

**Customary market size comments**

**Mortgage market structure**

**Prepayment modeling**

**Yield and OAS**

**Data and calibration**

**Interest rate models**

**Index projection**

**Monte Carlo analysis**

**Greeks**

**Validation**

**Robust parallelization**

**Summary**
MBS pool fine characteristics
Maturity distribution

<table>
<thead>
<tr>
<th>MTHS</th>
<th>#</th>
<th>BAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>350-359</td>
<td>46</td>
<td>7.0</td>
<td>61.0</td>
</tr>
<tr>
<td>360</td>
<td>23</td>
<td>4.5</td>
<td>39.0</td>
</tr>
<tr>
<td>TOTAL</td>
<td>69</td>
<td>11.4</td>
<td>100.0</td>
</tr>
</tbody>
</table>
### MBS pool fine characteristics

#### Age distribution

<table>
<thead>
<tr>
<th>AGE</th>
<th>#</th>
<th>BAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MTHS</td>
<td>LNS</td>
<td>$MIL</td>
</tr>
<tr>
<td>0</td>
<td>25</td>
<td>5.0</td>
<td>43.9</td>
</tr>
<tr>
<td>1</td>
<td>38</td>
<td>5.7</td>
<td>49.4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>.2</td>
<td>1.4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>.6</td>
<td>5.2</td>
</tr>
</tbody>
</table>

**TOTAL:** 69 | 11.4 | 100.0
MBS pool fine characteristics
Credit rating distribution

<table>
<thead>
<tr>
<th>FICO SCORE</th>
<th># LNS</th>
<th>BAL</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>500-549</td>
<td>1.2</td>
<td>2.0</td>
<td>64.00</td>
</tr>
<tr>
<td>550-599</td>
<td>13.7</td>
<td>14.9</td>
<td>72.00</td>
</tr>
<tr>
<td>600-649</td>
<td>30.5</td>
<td>46.4</td>
<td>78.00</td>
</tr>
<tr>
<td>650-699</td>
<td>18.2</td>
<td>25.6</td>
<td>82.00</td>
</tr>
<tr>
<td>700-749</td>
<td>6.1</td>
<td>10.0</td>
<td>85.00</td>
</tr>
<tr>
<td>750-799</td>
<td>1</td>
<td>1.2</td>
<td>95.00</td>
</tr>
<tr>
<td>TOTAL</td>
<td>69</td>
<td>11.4</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Harvey Stein

Mortgage OAS Analysis

Customary market size comments

Mortgage market structure

Prepayment modeling

Yield and OAS

Data and calibration

Interest rate models

Index projection

Monte Carlo analysis

Greeks

Validation

Robust parallelization

Summary
MBS pool fine characteristics

Servicers

<table>
<thead>
<tr>
<th>Servicer Name</th>
<th>#Loans</th>
<th>%Bal</th>
<th>WAC  Min/Max</th>
<th>WALA Min/Max</th>
<th>WARM Min/Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>GNMACMTGECORP</td>
<td>69</td>
<td>100.0</td>
<td>6.862</td>
<td>2</td>
<td>358</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.250/7.000</td>
<td>1/4</td>
<td>356/359</td>
</tr>
</tbody>
</table>
MBS pool fine characteristics

Sellers

Collateral Composition
FG A41492

Seller Distribution by % Balance as of Feb 2006

<table>
<thead>
<tr>
<th>Seller Name</th>
<th>#Loans</th>
<th>%Bal</th>
<th>WAC Min/Max</th>
<th>WALA Min/Max</th>
<th>WARN Min/Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>GNACMTCORP</td>
<td>PA 19044</td>
<td>189.100.0</td>
<td>6.862</td>
<td>6.250/7.000</td>
<td>1/4</td>
</tr>
</tbody>
</table>
MBS secondary market - TBAs

Pools are also sold before being created, trading as TBAs:

<table>
<thead>
<tr>
<th>MBS</th>
<th>TBA</th>
<th>MONITOR: BGN</th>
<th>'ASK'</th>
<th>Mtge</th>
<th>TBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>30YR GNSF 4.5</td>
<td>30YR GOLD 5</td>
<td>30YR FNMA 5</td>
<td>15YR FNCI 5</td>
<td>PAGE 1</td>
<td>OF 6</td>
</tr>
<tr>
<td>FEB 95-18</td>
<td>FEB 96-15</td>
<td>FEB 96-17</td>
<td>FEB 98-20</td>
<td>--TREASURY--</td>
<td></td>
</tr>
<tr>
<td>MAR 95-14</td>
<td>MAR 96-12</td>
<td>MAR 96-14</td>
<td>MAR 98-17</td>
<td>CT30T110-22 +03</td>
<td></td>
</tr>
<tr>
<td>APR 95-12</td>
<td>APR 96-10</td>
<td>APR 96-12</td>
<td>APR 98-16</td>
<td>CT101193-18 +03</td>
<td></td>
</tr>
<tr>
<td>FEB 98-10</td>
<td>FEB 98-27</td>
<td>FEB 98-25</td>
<td>FEB 100-15</td>
<td>CT5 198-28 +01</td>
<td></td>
</tr>
<tr>
<td>MAR 98-08</td>
<td>MAR 98-23</td>
<td>MAR 98-21</td>
<td>MAR 100-11</td>
<td>CT2193-19 --</td>
<td></td>
</tr>
<tr>
<td>APR 98-06</td>
<td>APR 98-20</td>
<td>APR 98-18</td>
<td>APR 100-09</td>
<td>CBE 14.49 --</td>
<td></td>
</tr>
<tr>
<td>FEB 100-08</td>
<td>FEB 100-29</td>
<td>FEB 100-29</td>
<td>FEB 100-19</td>
<td>--FUT/OTHER--</td>
<td></td>
</tr>
<tr>
<td>MAR 100-05</td>
<td>MAR 100-24</td>
<td>MAR 100-23</td>
<td>MAR 98-16</td>
<td>USA 4112-29 +01</td>
<td></td>
</tr>
<tr>
<td>APR 100-03</td>
<td>APR 100-21</td>
<td>APR 100-20</td>
<td>APR 98-14</td>
<td>TIA 108-07 +02</td>
<td></td>
</tr>
<tr>
<td>FEB 102-12</td>
<td>FEB 102-09</td>
<td>FEB 102-12</td>
<td>FEB 100-12</td>
<td>FVA 1105-16 +02</td>
<td></td>
</tr>
<tr>
<td>MAR 102-08</td>
<td>MAR 102-06</td>
<td>MAR 102-09</td>
<td>MAR 100-09</td>
<td>INDU 10749.7</td>
<td></td>
</tr>
<tr>
<td>APR 102-06</td>
<td>APR 102-03</td>
<td>APR 102-07</td>
<td>APR 100-07</td>
<td>FIDF 4.438</td>
<td></td>
</tr>
</tbody>
</table>

Australia 61 2 9777 0600  Brazil 5511 3040 4500  Europe 44 20 7333 7200  Germany 49 69 303410  Hong Kong 852 2877 2300  Japan 03 3581 8888  Singapore 65 6212 1090 40 295 105 6000  Copyright 2006 Bloomberg L.P. MHS-571-2 06-Feb-06 07:46:19
Pool analysis

Pool analysis is like mortgage analysis, except that it benefits from safety in numbers.

- Backed by a substantial number of individual loans, so variance is reduced.

While it used to be the case that this was at the cost of only knowing gross aggregate data about the pool, these days the fine structure of the pool is often disclosed as well, telling us:

- Location of individual loans,
- Size of individual loans,
- LTV,
- and credit ratings.

The only thing missing is individual borrower details.
The molecules — CMOs

In 1983, Solomon Brothers and First Boston created the first Collateralized Mortgage Obligation (CMO). They realized that more pools could be sold if the pool cash flows were carved up to stratify risk.

CMOs are:

- Backed by pools or directly by mortgages (whole loans), sometimes by as many as 20,000 of them.
- Split up cash flows of underlying collateral into a number of “bonds” or “tranches”.
- By creating desirable risk structures, tranches can be sold to a wider audience, and at a profit.

CMOs are essentially arbitrary structured notes backed by mortgage collateral.
Tranche types

Tranches vary by how the principal and interest are carved up.

- **Interest handling:**
  - Fixed cpn — Can behave like a pool or very differently, depending on how principal is paid.
  - POs — Only principal payments from underlying collateral.
  - IOs — Only interest payments.
  - Floaters — Where there’s a floater, there’s an inverse floater (when you have fixed rate collateral).
  - Inverse floaters.

- **Principal handling:**
  - Sequential Pay — Sequence of tranches. First gets principal until paid, then 2nd gets principal, etc. Last one is most prepayment protected and behaves most like an ordinary bond.
  - PACs — Scheduled principal will be payed as long as prepayment remains in a specified band.
  - TACs — Scheduled principal will be payed when prepayment is at a specified level.
  - Etc.
Example CMO

<table>
<thead>
<tr>
<th>Class</th>
<th>Orig Amt</th>
<th>Coupon</th>
<th>Orig WAL</th>
<th>Orig Maturity</th>
<th>CUSIP</th>
<th>GRADE</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CI</td>
<td>127,338</td>
<td>6.00</td>
<td>2.50</td>
<td>1/25/32</td>
<td>31394VRE2</td>
<td>--</td>
<td>SC, IO, NTL</td>
</tr>
<tr>
<td>DI</td>
<td>20,761</td>
<td>6.00</td>
<td>6.00</td>
<td>1/25/32</td>
<td>31394VRF9</td>
<td>--</td>
<td>SC, IO, NTL</td>
</tr>
<tr>
<td>FA</td>
<td>362,762</td>
<td>5.37</td>
<td>12.10</td>
<td>1/25/32</td>
<td>31394VRG7</td>
<td>--</td>
<td>SC, FLT, DLY, SUP</td>
</tr>
<tr>
<td>FI</td>
<td>96,555</td>
<td>4.93</td>
<td>4.00</td>
<td>1/25/32</td>
<td>31394VRH5</td>
<td>--</td>
<td>SC, IO, FLT, NTL</td>
</tr>
<tr>
<td>FO</td>
<td>96,555</td>
<td>0.00</td>
<td>4.00</td>
<td>1/25/32</td>
<td>31394VRJ1</td>
<td>12AC</td>
<td>SC, PO, AD, +</td>
</tr>
<tr>
<td>GS</td>
<td>11,277</td>
<td>13.56</td>
<td>17.50</td>
<td>1/25/32</td>
<td>31394VRK8</td>
<td>z32AB</td>
<td>SC, IO, NTL</td>
</tr>
<tr>
<td>IJ</td>
<td>76,208</td>
<td>6.00</td>
<td>16.30</td>
<td>1/25/32</td>
<td>31394VRL6</td>
<td>--</td>
<td>SC, IO, NTL</td>
</tr>
<tr>
<td>JD</td>
<td>76,208</td>
<td>0.00</td>
<td>16.30</td>
<td>1/25/32</td>
<td>31394VRM4</td>
<td>z32AB</td>
<td>SC, PO, PAC(11)</td>
</tr>
<tr>
<td>PM</td>
<td>124,568</td>
<td>5.00</td>
<td>6.00</td>
<td>1/25/32</td>
<td>31394VRN2</td>
<td>5DC</td>
<td>SC, PAC(11)</td>
</tr>
<tr>
<td>PN</td>
<td>153,898</td>
<td>6.00</td>
<td>8.00</td>
<td>1/25/32</td>
<td>31394VRP7</td>
<td>5ED</td>
<td>SC, PAC(11)</td>
</tr>
<tr>
<td>PR</td>
<td>104,883</td>
<td>6.00</td>
<td>11.00</td>
<td>1/25/32</td>
<td>31394VRQ5</td>
<td>7ED</td>
<td>SC, PAC(11)</td>
</tr>
<tr>
<td>PY</td>
<td>509,350</td>
<td>4.50</td>
<td>2.50</td>
<td>1/25/32</td>
<td>31394VRR3</td>
<td>2CC</td>
<td>SC, PAC(11)</td>
</tr>
<tr>
<td>SA</td>
<td>8,750</td>
<td>13.56</td>
<td>12.10</td>
<td>1/25/32</td>
<td>31394VRS1</td>
<td>z36EE</td>
<td>SC, INV, DLY, SUP</td>
</tr>
<tr>
<td>SC</td>
<td>10,203</td>
<td>13.56</td>
<td>6.10</td>
<td>1/25/32</td>
<td>31394VRT9</td>
<td>z19EE</td>
<td>SC, INV, DLY, SUP</td>
</tr>
<tr>
<td>SI</td>
<td>16,093</td>
<td>12.42</td>
<td>4.00</td>
<td>1/25/32</td>
<td>31394VRU6</td>
<td>16EE</td>
<td>SC, IO, INV, NTL</td>
</tr>
<tr>
<td>SO</td>
<td>16,093</td>
<td>0.00</td>
<td>4.00</td>
<td>1/25/32</td>
<td>31394VRV4</td>
<td>12AC</td>
<td>SC, PO, AD, +</td>
</tr>
<tr>
<td>ZA</td>
<td>100</td>
<td>6.00</td>
<td>10.40</td>
<td>10/25/33</td>
<td>31394VRU2</td>
<td>6EE</td>
<td>SC, Z, PAC(22)</td>
</tr>
<tr>
<td>CA</td>
<td>25,089</td>
<td>6.00</td>
<td>3.90</td>
<td>10/25/33</td>
<td>31394VRO0</td>
<td>2ED</td>
<td>SC, PAC(22)</td>
</tr>
</tbody>
</table>
Example CMO tranche

[Image of Bloomberg screen showing security description and tranche details]
# Example CMO collateral

![CMO collateral data](image)

## Bloomberg Group Description

- **FNR 05-118**
- **Group-1: REMIC**
- **Issuer: FANNIE MAE**

## CMO Collateral Details

### Group - Current

<table>
<thead>
<tr>
<th>Jan06</th>
<th>1,447,069,822</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net</td>
<td>6%</td>
</tr>
<tr>
<td>WAC</td>
<td>6.496%</td>
</tr>
<tr>
<td>WAM</td>
<td>25:0 300 mo</td>
</tr>
<tr>
<td>AGE</td>
<td>4:2 50 mo</td>
</tr>
<tr>
<td>Next Paymt</td>
<td>2/25/06</td>
</tr>
<tr>
<td>Rcd date</td>
<td>1/31/06</td>
</tr>
<tr>
<td>B.Median</td>
<td>PSA</td>
</tr>
<tr>
<td>PAC</td>
<td>66% SUP 26%</td>
</tr>
<tr>
<td>Beg Accrue</td>
<td>MIXED</td>
</tr>
</tbody>
</table>

### Group - Original

<table>
<thead>
<tr>
<th>Jan06</th>
<th>1,474,647,025</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net</td>
<td>6%</td>
</tr>
<tr>
<td>WAC</td>
<td>6.497%</td>
</tr>
<tr>
<td>WAM</td>
<td>25:1 301 mo</td>
</tr>
<tr>
<td>AGE</td>
<td>4:1 49 mo</td>
</tr>
<tr>
<td>1st Paymt</td>
<td>1/25/06</td>
</tr>
<tr>
<td>1st Settle</td>
<td>12/28/05</td>
</tr>
<tr>
<td>Px 160 PSA</td>
<td>11/30/05</td>
</tr>
<tr>
<td>PAC</td>
<td>66% SUP 27%</td>
</tr>
<tr>
<td>Dated</td>
<td>MIXED</td>
</tr>
</tbody>
</table>

### Monthly Payment

- Pays 25th day
- MIXED Delay
- Accrues 30/360

### Example CMO Collateral

- **PSA** - 317 372x
- **CPR** - 19.0 22.3x
- **WAM** - 300 301
- **WAC** - 6.50 6.50

**See Page 3 For Comments.**

---

**Note:** The image includes detailed financial data and information about a CMO collateral, which is a structured financial product that combines characteristics of mortgage-backed securities (MBS) and collateralized mortgage obligations (CMO). The data includes information on interest rates, payment dates, and other financial metrics crucial for understanding CMOs.
Computing the cash flows of a CMO tranche requires modeling the CMO, i.e. — converting the prospectus into a mathematical specification of the tranche payouts as a function of collateral cash flows.
CMO cash flow engine

The cash flow engine computes CMO cash flows for each scenario.

Inputs:
- CMO deal specification.
- Index projections.
- Current outstanding balance of each piece of collateral.

Given the above inputs, the cash flow engine parses and evaluates the deal specification, using the cash flows generated from running the prepayment model on each piece of collateral and amortizing it.
Prepayment modeling
Prepayment speeds

Prepayment speeds are to prepayment modeling what yield calculations are to interest rate modeling.

- **SMM** — Percentage of remaining balance above scheduled paid: 
  \[ 100 \frac{P_i' - P_i}{B_i} \] .

- **CPR** — SMM annualized: 
  \[ 100 \left( 1 - \left( 1 - \frac{\text{SMM}}{100} \right)^{12} \right) \] .

- **PSA** — “Prepayment Speed Assumption”:
  0.2% initially, increasing by 0.2% each month for the first 30 months, and 6.0% until the loan pays off. 200 PSA is double this rate, etc.

- **MHP** — PSA for manufactured housing (ABS, not MBS).

Looking at the value of a pool or CMO as a function of prepayment level is a useful analysis tool.
Prepayment speed graph
Prepayment modeling is a major component of MBS and CMO valuation. In some sense, CMO and MBS valuation is Monte Carlo analysis of the prepayment model.

In prepayment modeling:

- Salient features of prepayment are proposed.
- Evidence is collected statistically.
- Models are developed for these relationships.
Major prepayment components

- Housing turnover.
- Refinancing.
- Curtailment and Default.
Housing turnover

- Appears as relatively constant baseline level of prepayment = total (existing) home sales divided by total housing stock.
- Seasonality — Less movement in the winter.
- Seasoning — Chances of moving increase with age of mortgage, but tend to level off. A function of WAM, loan type, and prepayment incentive.
- Lock-in effect — High rates relative to mortgage coupon are a disincentive to moving when LTV is high.
- Rarely over 10% CPR.
Housing turnover illustration

Housing turnover behavior typically dominates prepayment when rates are low.

![Graph showing CPR over time for a 30-Year FNMA Orig 1998 Coupon 5.50, with lines representing Actual, HTurnover, and TotalPrepayment.](image)
Refinancing

Refinancing is the major interest rate dependent component.

- **“S-curve”** — Zero when rates are above the mortgage coupon, picks up as rates drop and tops out at some maximum level (not everyone can refinance).
- **Aging** — New loans less likely to be refinanced due to refi costs, but mitigated by high refi incentive.
- **Credit quality** — Lower credit quality less likely to refi. Can use level of cpn above mortgage rates at issuance in lieu of FICO score.
- **Burnout** — As borrowers refinance out of a pool, the remaining borrowers are less likely to refinance (unaware or unable).
- **Media effect** — Prepayments tend to surge around multi-year lows, presumably induced by press coverage encouraging refinancing.
- **Pipeline effect** — Prepayment rate spikes are asymmetric. Prepayment drops slower than it grew. Due to mortgage broker capacity limits causing refi applications to back up.
- **Lagged effects.**
Refinancing S-curve

Sample refi S-curves for normal credit and poor credit:
Curtailment and Default

Default results in return of outstanding principal when property is sold.

Curtailment is the reduction in maturity due to additional partial payment of principal. Without reamortization, results in additional principal pay downs on a monthly basis. Can also be from paying off remainder of an old mortgage.

- Tend to be independent of interest rates.
- Default risk grows and then drops as LTV decreases.
- Curtailment picks up towards end of mortgage life.
Curtailment and Default graph

These considerations lead to the following typical curtailment and default graph.
Loan level characteristics

Increased disclosure allows improved prepayment modeling.

Primary loan attributes:
- LTV
- FICO score
- loan size

Secondary loan attributes:
- occupancy
- property type
- loan purpose
Yield calculations
Yield calculations

Internal rate of return at a given price assuming a particular prepayment rate.

- Pick a prepayment speed.
- Generate cash flows.
- Solve for internal rate of return that gives quoted price.

Conceptually simple, but computationally intensive and involved when trying to value a CMO backed by 20,000 pools, with cash flows carved up across 100 tranches.
Yield example — PSA

YIELD TABLE

Yield example PSA 51 / 106
Yield example — BPM

**YIELD TABLE**

<table>
<thead>
<tr>
<th>Library</th>
<th>Fxd Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mtge</td>
<td>4.5700</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rate (bps)</th>
<th>2.32</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 BPM</td>
<td>+300 BPM</td>
</tr>
<tr>
<td>+200 BPM</td>
<td>+100 BPM</td>
</tr>
</tbody>
</table>

**AvgLife** 11.54, **Spread** 4.55, **Window** 3/25/06 - 7/25/06
OAS Analysis
From Yield to OAS

Consider a semi-annual bond with cash flows $C_i$ at times $t_i$ (with principal payment included in $C_N$) and (dirty) price $P$. When the bond is not callable, OAS is basically Z-spread, which is basically a spread over a set of bonds which is basically a difference in yields.

\[
P \approx \sum_{1}^{N} \frac{C_i}{(1 + \frac{Y_i}{2})^{2t_i}} \quad \text{Yield}
\]

\[
P \approx \sum_{1}^{N} \frac{C_i}{(1 + \frac{R+S}{2})^{2t_i}} \quad \text{Spread}
\]

\[
P \approx \sum_{1}^{N} \frac{C_i}{(1 + \frac{Y_i+S}{2})^{2t_i}} \quad \text{Z-Spread}
\]

\[
P \approx \sum_{1}^{N} \frac{C_i}{(1 + \frac{Y_i+S}{2})^{2t_i}} \quad \text{OAS}
\]
Optionality in OAS

When the bond has embedded optionality, OAS attempts to value the optionality. Doing this requires some assumptions regarding the evolution of interest rates:
MBS and CMO OAS

Compute average value under a large set of realistic scenarios. The OAS is the shift in the discount rates needed to arrive at a specified price.

Because of the complexity and path dependency of the prepayment model and the CMO tranche calculations, OAS analysis is done via Monte Carlo.

- Calibrate interest rate model.
- Generate interest rate scenarios — discount rates, longer tenor rates, par rates.
- Generate indices — current coupons, district 11 cost of funds, and whatever else is needed.
- For each scenario:
  - Compute prepayments for each piece of collateral.
  - Amortize each piece of collateral.
  - Compute tranche cash flows.
  - Discount cash flows at specified OAS to generate scenario price.
- Average of scenario prices is the price at the specified OAS.
Data and calibration
Data importance

Data:

- Most critical component of interest rate modeling.
- Weak model calibrated to good data is far better than a strong model calibrated to the wrong data.
- Data must have good pricing and capture risks of securities being valued.
- Characterizing risk in terms of related liquid instruments will help to answer the question of which interest rate model to use.
Discounting component

Traditionally, the treasury curve. Thought to be closest in risk to agency default risk (i.e. negligible), but:

- Heterogeneous — coupon bonds, some liquid and some illiquid.
- Sparse data — large gaps between maturities leaves interim pricing unknown (subject to interpolation method).
- Market price of conditional cash flows (i.e. option prices = Volatility data) unavailable.

Newer market standard — calibrate to the swap curve.

- Denser data — monthly at short end, annually further out. Less subject to interpretation.
- Homogeneous — cash rates and par rates, not bond prices at varying coupons. Mostly on a clean, nominal basis.
- Rich volatility data — ATM and OTM caps and floors are well known. ATM swaptions of various tenors and maturities are well known. Only OTM swaption pricing is subject to discussion.
- Hedging — swap market often used to hedge MBSs. Calibrating to it makes calculating appropriate hedges easier.
Volatility calibration

What to calibrate to?

- Calibrate to full volatility cube? Too expensive to hedge.
- Calibrate to one volatility? Which one?

Compromise 1: Two year and ten year swaptions are commonly used for hedging, so calibrate to them.

Compromise 2: Rather than calibrating to the entire smile, calibrate to the ATMs and pick a model which does a reasonable job of capturing the overall smile. This is also good because OTM swaption data is hard to come by.
Interest rate models
Popular short rate models

Log normal short rate models have been popular for many years in the fixed income markets.

- Simple and intuitive.
- A priori, rates shouldn’t go negative.
A log normal short rate model (LNMR)

One example is the first model we used for mortgage valuation. The short rate process is $r_t$, where:

$$\begin{align*}
    r_t &= e^{R_t + \theta_t} \\
    dR_t &= -a R_t dt + \sigma dW_t,
\end{align*}$$

with $a$ and $\sigma$ constants, $\theta_t$ is a function of time and chosen to calibrate the model to the discount curve. Under this model $R_t$ is a Gaussian process mean reverting to zero, and $r_t$ is log normal and mean reverting as well.
Drawbacks of 1 factor LNMR

- Model skew is flat in log normal terms, which is not what’s observed in the market today.
- Correlated rates.
- Calibration rigidity — can calibrate to the yield curve and two option prices. Mean reversion is known to be hard to calibrate. However, greater flexibility can be introduced with time dependent volatility (Black-Derman-Toy).
US skews 1 year out

It always pays to look at the data. Here are cap and swaption implied volatilities from the US market. The skew is substantial across maturities.
US skews 5 years out

The five year tenors are skewed as well.
How normal is it?

Fitting the CEV model \( dr = \sigma r^\beta dW \) to caplet skews shows that the US market is fairly close to normal \((\beta \text{ close to zero})\).
How normal is it?

Fitting errors for the CEV model to the US market aren’t too large, so above betas are reasonable.
The Linear Gaussian Markovian models (AKA Hull-White models) are a family of extremely tractable models. The two factor form is:

\[
\begin{align*}
    r_t &= \theta_t + X_t + Y_t \\
    dX_t &= -a_X X_t \, dt + \sigma_X(t) \, dW_1 \\
    dY_t &= -a_Y Y_t \, dt + \sigma_Y(t) \, dW_2 \\
    dW_1 dW_2 &= \rho \, dt
\end{align*}
\]
LGM advantages

- Fits overall historical behavior well with fixed mean reversion and correlation \( a_x = 0.03, \ a_y = 0.5, \ \rho = -0.7 \).
- Can be calibrated to two term structures of volatility.
- Very tractable — Don’t underestimate the value of closed form solutions (or decent approximations) for bond prices, caplet prices, and swaption prices and swap rates as a function of \( (X_t, Y_t) \).
- Much closer to market skew than log normal models.
LGM behavior

- High mean reversion factor dampens long term effect of that parameter, leaving long tenor volatility mostly determined by low mean reversion factor.
- Both factors impact short tenor volatility.
- Negative correlation reduces volatility of short maturities relative to long maturities and allows this behavior to persist in time.
Market volatility term structure

Swaption term structure — implied volatilities as a function of maturity for various tenors.
LGM volatility term structure

LGM model's swaption term structure — implied volatilities produced by the model, as a function of maturity for various tenors.
Index projection
Index projection problem

- Prepayment model needs appropriate inputs (such as monthly mortgage refi rates).
- Floater indices (such as D11COFI) need to modeled.
- Neither need be discount rates, and hence, can’t be read directly from model.
Index projection solutions

- Not a problem for discount rates (LIBOR, swap rates, etc). Compute from model:
  - For one factor log normal — build lattice and compute future implied long tenor rates for each month as a function of the short rate.
  - For 2 factor LGM — rates are given by formulas.
- Avoid problem — calibrate prepayment model directly to discount rates (LIBOR, swap rates, etc).
- Simplify — to first order can ignore volatility of index relative to discount rates. How different can refi rates be on two different dates that have the same swap curve? Regress and model as a function of the rates. Or get fancy and take vol into account as well.
Chosen solution

We chose simple:

- Treat implied rates as a function of swap rates.
- Regress — proxy the current coupon for each collateral type as a weighted average of the 2 year and 10 year rates.
- Adjust for current actual value of current coupons.
- Adjust in prepayment model for fact that a proxy for the refi rate is being used.
- Potentially use lagged data (e.g. - D11COFI in ARM Wrestling: Valuing Adjustable Rate Mortgages Indexed to the Eleventh District Cost of Funds, by Stanton and Wallace.)
Monte Carlo analysis
The need for speed

With 20,000 pieces of collateral, interpreted rules for paying out cash flows and time consuming prepayment projection calculations, running the 100,000 scenarios necessary for accurate valuation will take some time. If a scenario takes 0.0001 seconds on one piece of collateral, then we’ll have to wait 2.3 days. 100 scenarios would take 3.3 minutes.

So, it pays to reduce the number of scenarios needed.
In the one factor case, the method we developed is the bifurcation tree approach.

- Begin with two paths starting at the current short rate.
- Evolve them to match the moments of the model relative to the last common point.
- Periodically allow the paths to split:

\[
R_u = \mathbb{E}[R_t | R_b = R] + \sqrt{\text{Var}[R_t | R_b = R]}
\]

\[
R_d = \mathbb{E}[R_t | R_b = R] - \sqrt{\text{Var}[R_t | R_b = R]}
\]
E\left[ R_t \mid R_b = R \right] \text{ and } \text{Var}\left[ R_t \mid R_b = R \right] \text{ can be computed from the SDE for } R_t \text{ by using } e^{at} \text{ as an integrating factor, yielding:}

\begin{align*}
R_t &= e^{a(b-t)} R_b + \int_b^t \sigma e^{a(s-t)} dW_s,
\end{align*}

so

\begin{align*}
E\left[ R_t \mid R_b \right] &= R_b e^{-a(t-b)},
\end{align*}

and

\begin{align*}
\text{Var}\left[ R_t \mid R_b \right] &= \frac{\sigma^2}{2a} \left( 1 - e^{-2a(t-b)} \right).
\end{align*}

1 factor LNMR
The end result, with 5 selected bifurcation maturities, is the following tree:
Bifurcation tree behavior

Advantages:

- Reasonably straight forward.
- Conditional mean and variance of underlying $R$ process is matched at bifurcation points.
- Results look reasonable with fairly small numbers of paths.

Disadvantages:

- Variability is coarse (semiannual steps).
- Unclear that overall variance is captured.
- Doesn’t extend well to the multi-factor case.
We’ve investigated a variety of variance reduction techniques for the 2 factor LGM model.

The first is to recognize that the choice of numeraire has a major impact on Monte Carlo efficiency.

Consider valuation of a price process $V$ under two different numeraires $N$ and $N'$, and corresponding equivalent Martingale measures $Q$ and $Q'$.

$$ V_0 = N_0 E^Q \left[ \frac{V_t}{N_t} \right] = N'_0 E^{Q'} \left[ \frac{V_t}{N'_t} \right] $$
The relationship between \( N, N', Q \) and \( Q' \) is that

\[
\frac{N}{N_0} = \frac{dQ}{dQ'}.
\]

where \( dQ/dQ' \) is the Radon-Nikodym derivative of \( Q \) with respect to \( Q' \).

So, a numeraire that’s higher on high rates will have an equivalent Martingale measure that’s correspondingly higher as well, and thus the Monte Carlo associated with the higher numeraire will sample more heavily from this region.
LGM — VR1 — Numeraire considerations

MBS gross behavior:

- High rates — Low prepayment — MBS behavior becomes more stable.
- Low rates — High prepayment — MBS behavior more interesting.

Numeraire selection — Choose numeraire that’s low for high rates and high for low rates:

- Integrated form of LGM is very convenient — simplifies formulas. But, numeraire is low for low rate paths, causing poor Monte Carlo behavior.
- Standard money market numeraire is much better. Harder to work with, but does better importance sampling.
LGM — VR2 — path shifting

Adjust paths so that sample at least captures yield curve.

- Similar to doing control variate on the discount rates.
- \( r_t = \theta_t + X_t + Y_t \), and \( X_t \) and \( Y_t \) are Gaussian and symmetric around zero, so just compute a new \( \theta \) to get discounting correct for chosen path set.

VR1 + VR2 yields pricing standard deviation of about 1bp for 2,000 paths, or \( \leq 1 \text{ bp error} \approx 30\% \) of the time, and \( \leq 2 \text{ bp error} \approx 66\% \) of the time.
LGM — VR3 — PCA

- Randomly sample paths $P = (X_{t_1}, \ldots X_{t_n}, Y_{t_1}, \ldots Y_{t_n})$
- $X_{t_i}$ and $Y_{t_i}$ are jointly distributed as a $2n$ dimensional Gaussian with zero mean.
- Let $C$ be the covariance matrix of these $2n$ random variables.
- If $Z = (z_1, \ldots, z_{2n})^t$, and the $z_i$ are IID $N(0, 1)$, and $C = AA^t$, then $AZ$ has the same distribution as $P$.
- Choose $A$ to be the matrix whose columns are the eigenvectors of $C$, scaled by the square roots of their eigenvalues. Then $C = AA^t$.
- The best $k$ factor approximation to $P$ is given by using the first $k$ columns of $V$.
- Small number of vectors capture most of variance.
- In 1 factor LGM — 7 vectors out of 360 for 95% of variance.
- In 2 factor LGM — 9 vectors out of 720 for 95% of variance.
LGM — VR4 — Weighted PCA

- Most of action in pool is up front, both because of prepayment and because of discounting.
- Weight PCA with $e^{-\alpha t}$ to effect this.
- Weight two factors differently as well (High MR factor doesn’t have as big an impact on MBS pricing as low MR factor).
LGM — VR5 — Local antithetic sampling

- Pure antithetic just makes sure sample mean is correct.
- Not so effective in interest rate MC.
- Do local antithetic instead (”uniform sampling with antithetic noise” (UWAN)) - Dupire and Savine:
  - Build a grid.
  - Pick antithetic pairs from each box.
  - Uses MC to eliminate convexity caused bias.
- Done on first few eigenvectors.
LGM — VR1+VR2+VR4+VR5 —
Weighted PCA with local antithetic sampling

Combine methods:

- VR1 — Money market numeraire.
- VR2 — Path shifting.
- VR4 — Weighted PCA.
- VR5 — Local antithetic sampling.

VR1+VR2+VR4+VR5 = standard deviation of 300 paths is lower than 5000 for VR1+VR2.
Adding Sobol sequences to the mix:

- Use Sobol sequences to randomly scale eigenvectors.
- Further reduces variance.
Results

![Graph showing RMSE vs Number of Paths](image)

**Legend:**
- FWD
- PC UWAN1
- PC UWAN2
- PC UWAN3
- PC Sobol
- WPC UWAN1
- WPC UWAN2
- WPC UWAN3
- WPC Sobol
Greeks
Greek errors

Greeks magnify pricing errors.

\[
\frac{dP}{dS} \approx \frac{P(S + h) - P(S - h)}{2h}
\]

If \( P(S) \) is the true model price, and \( \tilde{P}(S) \) is what we compute, then the error is \( \epsilon(S) = \tilde{P}(S) - P(S) \).

\[
\frac{\tilde{P}(S + h) - \tilde{P}(S - h)}{2h} = \frac{P(S + h) + \epsilon(S + h) - (P(S - h) + \epsilon(S - h))}{2h}
\]

\[
= \frac{\Delta P/2h + \Delta \epsilon/2h}{2}
\]

\[
= dP/dS + \frac{d^3P/dS^3}{3!} h^2 + \ldots + \Delta \epsilon/2h
\]

So, the error in the calculation is the error from dropping the higher order terms and from the change in error with respect to \( S \).
Greek errors

Errors in derivative are caused by:

- High order terms corrupting finite difference ("convexity").
- Pricing error.

Error control:

- Small $h$ reduces Greek error from convexity.
- Large $h$ reduces Greek error from pricing error.

Pricing error issues:

- Finite difference — $\epsilon$ flat, minimal problems.
- Monte Carlo — $\epsilon$ large and random — error goes to $\infty$ as $h \to 0$.

Solution for Monte Carlo:

- Make $\epsilon$ less random — Use the same paths, or the same random number seed.
- Accurate duration with 25bp shift, even when pricing variance is as large as 6bp.
Which Greeks?

To compute duration, rates are shifted while other inputs are held constant. How should the option data be held constant?

- Hold prices constant — Doesn’t make sense. Option prices have to change as rates make them more or less in the money.
- Hold vol constant — Vol is a log normal vol, but LGM model is normal. If vol is held constant, then model volatility will change when the rates are shifted.
- Hold *normal* vol constant — Most consistent with LGM model.

The situation between normal vol and log normal vol is reversed for the LNMR model.
Validation
Validation

How to make sure entire process works?

General methods:

- Results are stable over time. Jumps can be explained by occurrence of a change in the market.
- Fitted parameters stable over time as well.
- Prices and OASs behave appropriately
  - Flat in neighborhood of current cpn.
  - Expected relationships between different securities hold.
  - Prices and OASs move as expected when input parameters are changed:
    - Yield curve shifts.
    - Volatility shifts.
    - Shifts of current index values.

Mortgage specific:

- Compare to empirical durations.
Robust parallelization
Parallelization

Variance reduction alone is insufficient to compute OASs in real time. Need parallelization

- Linux clusters.
- 6 clusters, 50 dual CPU PCs each = 100 CPUs per cluster
- Embarrassingly parallel sometimes isn’t
  - Communication costs to farm out results and get back can render parallelization useless.
  - Old cluster - 750mhz, 100mbps ethernet — 60 seconds, 6 in parallel.
  - New cluster - 3.0ghz, 100mbps ethernet — 15 seconds, 4 seconds in parallel.
- Data dissemination problem — 2 gb of deal and collateral specifications.
- PCA parallelization — compute time vs communication speed.
Request flow

- User hits `<go>`
- Computation data is assembled:
  - current values
  - user selected values.
- request sent to dispatcher
- dispatcher queues until an idle server is available. Retries failed requests.
- Server receives request.
- Explodes base calculation into individual path requests
- farms them out across the cluster
  - redundancy and robustness
- assembles the result
- farms out remaining requests
- assembles results
- replies to client.
Data dissemination

How to keep 2gb current on 300 machines?

- Layered approach.
- Source keeps head machines up to date.
- Head machines update remainder of cluster.
- Distribute within cluster in a tree fashion:
  - Copy from head to 4 children.
  - Each time a node is updated, it starts updating 4 new nodes, and its updater starts updating a new node.
  - Utilizes full bandwidth of ethernet switch — 100mbps from each machine to switch, but independent pairs of machines can sustain this up until the switch’s backplane capacity.
Parallelization robustness

Preventing machine problems from causing calculation failures.

Errors encountered:
- Unstripable curves.
- Overheating machines.
- Flakey hard disks.
- Data unavailable.
- Bad data supplied.

Layered approach:
- Requests -> dispatcher -> OAS server -> slaves.
- If dispatcher gets an error (on a full request), it resends (up to the retry limit).
- If OAS server gets an error (on a path), it marks the slave as bad and tries again. If it gets confirmation of the error, the slave is marked good and the path is listed as bad. If not, the slave is no longer used.
Even embarrassingly parallel problems might have trouble parallelizing.

- Even the simplest variance reduction adds up startup costs and communication overhead.
- If startup costs can’t be distributed as well, then they yield a hard limit on parallelization speedup.
- PCA analysis is slow enough that it’s hard to make it actually save time.
  - Compression of data being distributed.
  - Tree distribution of data.
  - Optimize PCA.
  - Parallelize PCA.
  - Partial PCA.
Summary

CMO valuation is big science — lots of moving parts, with each one drawing on a different area:

- **Prepayment modeling:**
  - Statistical validation and modeling of economic and behavioral analysis.

- **Data selection:**
  - Risk analysis.

- **Interest rate modeling:**
  - Classic arbitrage pricing theory.

- **Index projection:**
  - Statistical analysis.

- **Monte Carlo analysis:**
  - Numerical methods.
  - Variance reduction techniques.

- **Parallelization:**
  - Building computation clusters.
  - Analysis and optimization of parallel algorithms.

As Emanuel Derman says, the best quants are interdisciplinarians. CMO valuation is one area that requires it.