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Counterparty Risk, CVA, and Basel III

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Outline

Need for CVA

We live in an increasingly risky world.

- Bank failures.
- Global recession.
- Libor rates far from the "risk free" rate.

Pre-crisis, swaps often traded without counterparty risk being taken into account. Today, the counterparty could be a low quality bank.

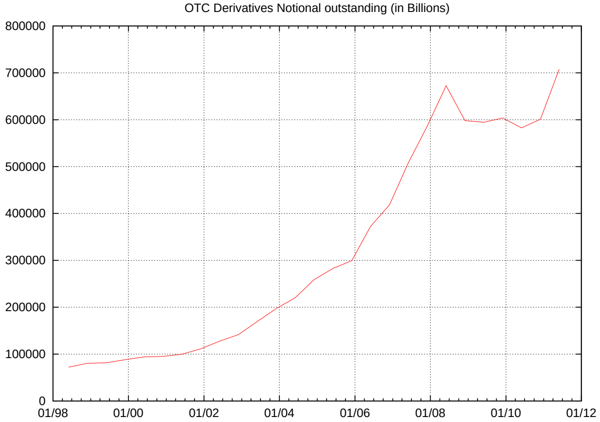
Size

Despite the financial crisis, the OTC derivatives markets continue to be a big part of the market, and interest rate derivatives are the largest part of the OTC derivatives market.

- OTC derivatives notional outstanding
  - \$547 trillion in December 2008
  - 70% in interest rate derivatives
  - \$605 trillion in June 2009
- OTC derivatives gross market value
  - June 2008 — \$20 trillion
  - December 2008 — \$32 trillion — up 60%
- IRD gross market value
  - June 2008 — \$9 trillion
  - December 2008 — \$18 trillion — doubled in 6 months.

And despite continued market turmoil, the derivatives market has continued to grow.

Notional outstanding



Demand

There is much demand for managing counterparty risk

- Accounting standards — FASB 157 (now "Topic 820, Section 10") and IAS 39 — credit risk must be taken into account.
- Regulators.
- Risk managers.

The IASB even issued a request for comment on counterparty risk calculation methodologies.

While the Dodd-Frank Act pushes OTC derivatives into clearing houses, thus mitigating counterparty risk, but:

- Risk doesn't go away — clearing houses will be bearing and collateralizing for counterparty risk.
- Corporations may still remain off of clearing houses.
- Collateralization costs must be computed and are similar to counterparty valuation adjustments.

Counterparty risk

Counterparty risk — the exposure to loss due to a specific counterparty failing to meet contractual obligations, i.e. defaulting.

Often restricted to counterparties on OTC derivatives contracts, but doing so is myopic. If a counterparty defaults, all of their contracts are affected:

- OTC derivatives
- bond issues
- stock issues
- debts, loans, ...

Compartmentalization of risks must be based on the type of risk, not the type of security.

So, it's worthwhile to think of counterparty risk as the impact of default risk.

Bond counterparty risk

Example — Investor is long a bond.

- Counterparty is issuer.
- Issuer defaults — investor loses bond cash flows, but gets recovery on the bond.
- Recovery is a percentage of the *principal* of the bond.
- Default causes loss of interest payments, and early (but partial) return of principal.
- Could be an improvement if bond is trading at a sufficient discount.

Example — Investor is short a bond.

- No counterparty risk.

Swap counterparty risk

Example — Investor enters into a swap.

- Counterparty is the entity with which the swap was transacted.
- Investor is simultaneously long one leg of the swap, and short the other leg.
- Counterparty defaults — investor loses swap cash flows, but gets recovery on the swap.
- Recovery is a percentage of the *market value* of the swap.
- If swap value is positive then there's a loss.
- If swap value is negative, then there is *no loss*.

Risk modifications — Netting

Netting agreements:

- Optional part of the ISDA master agreement.
- In the event of default, recovery is on the net market value of all contracts covered by the agreement.

No netting agreement:

- Investor owns two 5 year 5% swaps to counterparty — one is pay fixed, the other is receive fixed.
- No net market exposure — the two positions cancel out.
- Substantial counterparty exposure:
  - Get recovery on the market value of the positive swap
  - Still owe full value on the market value of the negative swap

With netting agreements, exposure at any given time is on the net market value of all securities covered. Risk is reduced.

Risk modifications — Collateralization

Collateralization

- ISDA credit support annex (CSA).
- At the end of the period (day, week, etc), if swap value exceeds a threshold, collateral must be posted.
- Exposure is to the threshold plus the movement of the market value over the period.

- Exposed to threshold plus the market moves between default and liquidation.
- Leads to requirement of initial margin from clearing houses.

Unintended consequences

Risk modifications change effective seniority structure of debt.

- Netting — contracts which are assets are used to pay some contracts which are debts in advance of other debts.
- Collateralization — cash and other assets of firm used as collateral are used to pay corresponding contracts in advance of other debts.
- Reduces value to bond holders and other creditors.

So, to a certain extent, using risk mitigants on derivatives debt is really transfering the risk to the other creditors.

Risk mitigation and clearing houses

Clearing houses mitigate risk via netting, collateralization, and reassignment of contracts.

- Initial margin plus daily variational margin attempts to make needed cash available in the event of a default.
- If that proves insufficient, coverage comes from backers of clearing house and equity of the firm itself.
- Netting is a two edged sword — with one clearing house, there's definitely increased netting. With multiple clearing houses and some structures not on clearing houses, it's unclear if netting is increased.
- Reassignment of contracts prevents losses due to market impact. But in a real crisis, reassignment might fail.

Risk mitigation clauses

Another approach to risk mitigation that is becoming popular is to include risk mitigation clauses (A.K.A. Additional Termination Events):

- Contract can be closed out at replacement value if the counterparty's rating drops.
- Contract can be closed out at market value prior to maturity.

Issues:

- What exactly is "replacement value"?
- How is closing out a swap early any different from entering into the reversing swap and novation?
- By forcing closeout at a rating change, are you decreasing counterparty risk while increasing systemic risk?
- Will this really work at the next crisis — what if the market is illiquid?

Counterparty Valuation Adjustments

How does the counterparty exposure and the risk of default impact the value of the security?

- The Credit Valuation Adjustment (CVA) is the cost of the potential loss.
- Risk free price - CVA = price of risky security.

Counterparty risk — long only vs long/short

Counterparty risk calculations are far more complicated for instruments that are a combination of long/short positions than for long only instruments.

- In a long only instrument, (like a bond position), counterparty risk can be judged by using models that can incorporate a discount curve shift.
- In a long/short instrument, (like a swap position), the instrument can potentially be an asset or a liability. When it's an asset, default results in a loss. When it's a liability, default results in no change.

Bond counterparty risk

If a bond with a coupon of C pays f times per year at times t_i, with maturity t_n, the value of the bond is:

$$\sum^{n} \frac{C}{f} D(t_i)S(t_i) + 100D(t_n)S(t_n) + \int^{t_n} 100RD(t)P(t)dt,$$

- $P(t)$ — default probability density function.
- $S(t) = 1 - \int^{t} P(s)ds$ — survival probability for time t (the probability of no default before time t).
- R — bond recovery rate.
- $D(t)$ — risk free discount factor for time t.

This is the standard CDS model applied to a fixed coupon bond (details in the Appendix). Note that it assumes independence of rates and default.

Equivalent par curve

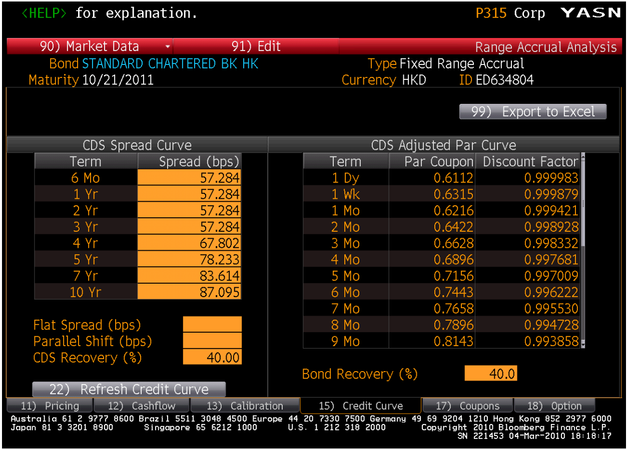
To get a feel for the impact of credit spreads on bond values, we can compute the par curve for the risky bond. For $t_i = i/f$, and for each n, solve for $C(t_n)$ such that

$$100 = \sum^{n} \frac{C(t_n)}{f} D(t_i)S(t_i) + 100D(t_n)S(t_n) + \int^{t_n} 100RD(t)P(t)dt,$$

Then $C(t_n)$ is the implied par curve — the coupons that the issuer with this CDS spread curve would theoretically use to issue debt at par.

Equivalent par curve example

On the Bloomberg terminal, we do this calculation in YASN - the structured notes calculation screen.



<HELP> for explanation. P315 Corp YASN

90) Market Data 91) Edit Range Accrual Analysis

Bond STANDARD CHARTERED BK HK Type Fixed Range Accrual

Maturity 10/21/2011 Currency HKD ID ED634804

99) Export to Excel

CDS Spread Curve

Term	Spread (bps)
6 Mo	57.284
1 Yr	57.284
2 Yr	57.284
3 Yr	57.284
4 Yr	67.802
5 Yr	78.233
7 Yr	83.614
10 Yr	87.095

Flat Spread (bps)

Parallel Shift (bps)

CDS Recovery (%) 40.00

22) Refresh Credit Curve

CDS Adjusted Par Curve

Term	Par Coupon	Discount Factor
1 Dy	0.6112	0.999983
1 Wk	0.6315	0.999879
1 Mo	0.6216	0.999421
2 Mo	0.6422	0.998928
3 Mo	0.6628	0.998332
4 Mo	0.6896	0.997681
5 Mo	0.7156	0.997009
6 Mo	0.7443	0.996222
7 Mo	0.7658	0.995530
8 Mo	0.7896	0.994728
9 Mo	0.8143	0.993858

Bond Recovery (%) 40.0

11) Pricing 12) Cashflow 13) Calibration 15) Credit Curve 17) Coupons 18) Option

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2010 Bloomberg Finance L.P.
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Recovery rate impact

Flat curves and equal recovery rates give an equivalent par curve roughly equal to the swap curve shifted by the CDS spreads.

Otherwise, there can be significant differences, as we see here for a flat 100bp CDS curve and a flat 3% swap curve.

CDS recovery	0%	40%	80%	40%
Bond recovery	0%	40%	80%	0%
Term				
1 Wk	4.2216	4.2241	4.2366	5.0783
1 Mo	4.0041	4.0071	4.0223	4.7141
6 Mo	3.9861	3.9948	4.0385	4.6741
1 Yr	4.0206	4.0361	4.1144	4.7196
2 Yr	4.0333	4.0420	4.0857	4.7252
3 Yr	4.0333	4.0419	4.0855	4.7251
5 Yr	4.0322	4.0408	4.0845	4.7233
7 Yr	4.0321	4.0407	4.0843	4.7231
10 Yr	4.0316	4.0403	4.0839	4.7223

Par curves vs default probabilities

Once the risky par curve is computed, one is tempted to strip it and use it for discounting.

Good points:

- This is simple, straight forward, and in line with common practices.
- This will properly price par bonds back to par.
- If the bond recovery rate is zero, this properly prices *all* bonds!

In the zero recovery rate case, this makes the risky discount factors

$$D(t_i)S(t_i),$$

so the risky spot rate curve $\bar{R}$ is given by

$$\bar{R}(t_i) = -\log(D(t_i)S(t_i))/t_i = R(t_i) - \log(S(t_i))/t_i,$$

where R is the risk free rate. So, the survival probabilities add a spread of $-\log(S(t_i))/t_i$ (the average hazard rate) to the risk free rate. This spread is roughly the CDS spread, adjusted by the CDS recovery rate.

Par curves vs default probabilities

Using the risky par curve directly for calculations also has drawbacks.

- For nonzero recovery rates, prices produced by this method on non-par bonds will differ from the default based method.

There is a difference between the two methods when the bond is a couple of hundred basis points away from par (roughly 5 to 15 basis points for a 100 bp CDS spread), but given other general uncertainties (such as the recovery rate, or the spread between bonds and CDS), this is a reasonable margin of error, and can be folded into OAS adjustments.

Bottom line — adding a spread to the risk free curve is a reasonable way to price risky bonds.

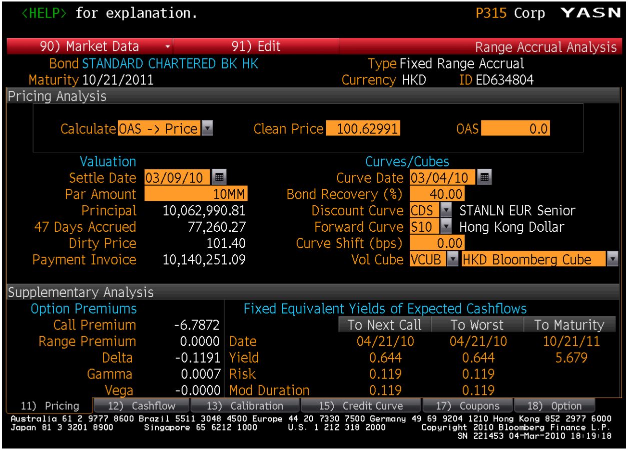
Par curves and OAS

If the spread curve is flat, it roughly amounts to a shift of the par curve, which is roughly adding an OAS.

Using the risky par curve in such a calculation is an improvement, in that it factors in the shape of the CDS spread curve.

Advanced risky bond calculations

Once we have a risky par curve, we can use it to apply risky spreads on top of a risk free interest rate derivatives models to analyze risky bonds with embedded options:



Counterparty risk in swaps — Characteristics

Consider a five year swap in a flat interest rate environment.

There is volatility dependent risk:

- Zero volatility:
  - Swap is always nearly zero market value.
  - Minimal default risk.
- High volatility:
  - Swap value in future can be substantial.
  - Potentially substantial loss upon default.

There is curve dependent risk:

- In a steep interest rate environment, the swap is expected to be heavily off the money for the duration of its life.
- Far more counterparty risk.

Counterparty risk in swaps — Valuation

Let $V(t)$ be the value of the risk-free swap at time t, and let R be the recovery rate on the underlying swap.

If the counterparty defaults at time τ, the payoff for holding the swap is:

- If $V(\tau) > 0$ we get $R \times V(\tau)$.
- If $V(\tau) < 0$ we still owe $V(\tau)$.

The above payoff is

$$\begin{aligned} &R\max(V(\tau),0) + \min(V(\tau),0) \\ &= V(\tau) - (1-R)\max(V(\tau),0) \end{aligned}$$

The quantity $\max(V(\tau),0)$ is the payoff of an option to enter into what's left of the swap at time τ — i.e. - a swaption maturing at time τ.

Counterparty risk in swaps — Valuation

The loss at default time τ is:

$$(1 - R)\max(V(\tau), 0)$$

The cost of this loss is:

$$\begin{aligned}
\mathrm{CVA} &= N(0)E[(1-R)\max(V(\tau),0)/N(\tau)1_{\tau<T}] \\
&= (1-R)N(0)E[\int_0^T \max(V(t),0)/N(t)\delta(t-\tau)dt] \\
&= (1-R)N(0)\int_0^T E[\max(V(t),0)/N(t)\delta(t-\tau)]dt
\end{aligned}$$

where the expectation is the equivalent martingale measure with respect to the numeraire N.

Counterparty risk in swaps — Valuation

If $\max(V(t), 0)/N(t)$ (the value of the call) and $\delta(t-\tau)$ (the default event) are independent, then the expectation factors and the CVA is:

$$
\begin{aligned}
\mathrm{CVA} &= (1-R)N(0)\int_0^T E[\max(V(t),0)/N(t)\delta(t-\tau)]dt \\
&= (1-R)\int_0^T N(0)E[\max(V(t),0)/N(t)]E[\delta(t-\tau)]dt \\
&= (1-R)\int_0^T S(t)P(t)dt
\end{aligned}
$$

where $S(t)$ is the current value of the swaption to enter into the remainder of the swap at time t, and $P(t)$ is the default time probability density function.

CVA subtleties

CVA valuation can be a little subtle. First of all, it's impractical to compute the above integral. One approach is discretization. Divide the time interval $[0, T]$ into periods $[t_i, t_{i+1}]$, and select $\bar{t}_i \in [t_i, t_{i+1}]$. Then

$$\begin{aligned} \mathrm{CVA} &= (1-R)\int_0^T S(t)P(t)dt \\ &\approx (1-R)\sum S(\bar{t}_i)\bar{P}(t_i) \end{aligned}$$

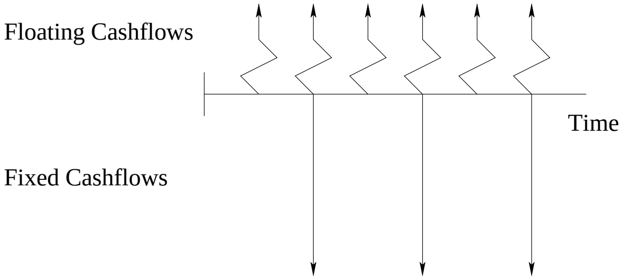
where $\bar{P}(t_i) = \int_{t_i}^{t_{i+1}} P(t)dt$ is the probability of defaulting in interval $[t_i, t_{i+1}]$.

The finer one subdivides the time period, the more accurate the calculation. We've found that it suffices to divide the time interval according to the cashflows of the swap as long as one values the call options at the midpoints of the intervals.

CVA subtleties

Another issue is accurately valuing $S(\bar{t}_i)$. This is the value of the option to enter into the tail of the swap, which is slightly different from the swaption maturing at time $\bar{t}_i$.

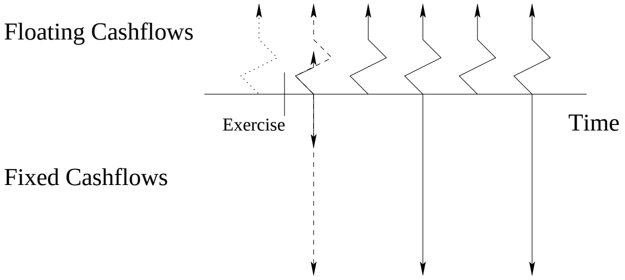
Swap cash flows:



CVA subtleties

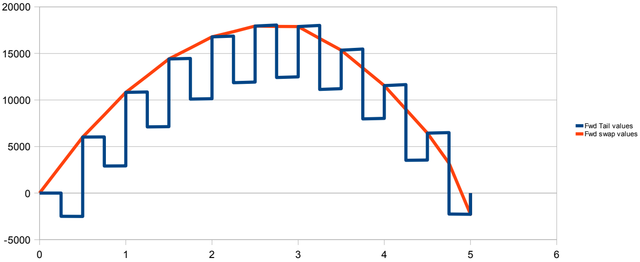
When exercising a swaption to enter into a swap with an odd first coupon, the first coupon is adjusted — the floating leg references a shortened index, and the fixed leg only accrues over the remainder of the period.

However, in the event of default at that time, the full cash flows are lost.



CVA subtleties

The difference between the value of the forward start swap and the corresponding tail of the underlying swap will typically be substantial, as is illustrated in the following graph of the two for a 5 year at the money payer swap, on a 1 million notional in a rising interest rate environment.



Forward swap rates

Consider the pay fixed swap with fixed rate F that the holder of a swaption would receive on exercise (at time t).

Let the underlying floating (fixed) leg pay at times t_i (t'_i). Then the time t value of the swap is[1]

$$S(t) = \sum L(t, t_i, t_{i+1}) Z(t, t_{i+1}) \alpha_i - \sum F \alpha'_i Z(t, t'_i),$$

Since $t \leq t_1$,

$$L(t, t_i, t_{i+1}) = (1/\alpha''_i)(Z(t, t_i)/Z(t, t_{i+1}) - 1)$$

(with Libor accrual factor α''_i), so if we assume $\alpha_i = \alpha''_i$, then $S(t)$ can be written in terms of Z as:

$$S(t) = Z(t, t_1) - Z(t, t_n) - \sum F \alpha'_i Z(t, t'_i),$$

This gives us the standard expression for the time t value of a forward start swap.

[1] $L(w, x, y)$ is the time w forward Libor rate setting at time x and paying at time y $Z(x, y)$ be the time x price of a zero coupon bond paying at time y, and α_i (α'_i) are the accrual fractions for floating (fixed) payment periods t_i to t_{i+1} (t'_i to t'_{i+1}), respectively.

Tail swap rate

The tail of the existing swap is slightly different. Its cash flows and accruals are the same as for the forward swap, except for the first period, where the reset date ($\bar{t}_1$) is earlier than the valuation time t, and the accrual factors correspond to the full period rather than the partial period. The time t value of the first floating cash flow is

$$\frac{Z(t,t_2)}{Z(\bar{t}_1,t_2)} - Z(t,t_2),$$

so the value of the tail is

$$\begin{aligned}
\bar{S}(t) &= \frac{Z(t,t_2)}{Z(\bar{t}_1,t_2)} - Z(t,t_2) + \sum_2 L(t,t_i,t_{i+1})Z(t,t_{i+1})\alpha_i \\
&\quad -F\bar{\alpha}'_1 Z(t,t_1) - \sum_2 F\alpha'_i Z(t,t'_i) \\
&= \frac{Z(t,t_2)}{Z(\bar{t}_1,t_2)} - Z(t,t_n) - F\bar{\alpha}'_1 Z(t,t_1) - \sum_2 F\alpha'_i Z(t,t'_i).
\end{aligned}$$

Forward swaps and tail swaps compared

The difference between the values of the two is

$$\bar{S}(t) - S(t) = \frac{Z(t, t_2)}{Z(\bar{t}_1, t_2)} - Z(t, t_1) - F(\bar{\alpha}_1' - \alpha_1')Z(t, t_1).$$

This is the adjustment that needs to be made to convert the swaption payoff to the payoff of the option on the tail swap.

Forward swap vs tail

Under the equivalent martingale measure with respect to numeraire N, the time zero value of the forward start swaption is

$$N(0)E_0[\max(S(t),0)/N(t)]$$

Instead of this, we need the value of the option on the tail of the swap. Concentrating on the payoff, if

$$D = \bar{S}(t) - S(t) = \frac{Z(t,t_2)}{Z(\bar{t}_1,t_2)} - Z(t,t_1) - F(\bar{\alpha}'_1 - \alpha'_1)Z(t,t_1)$$

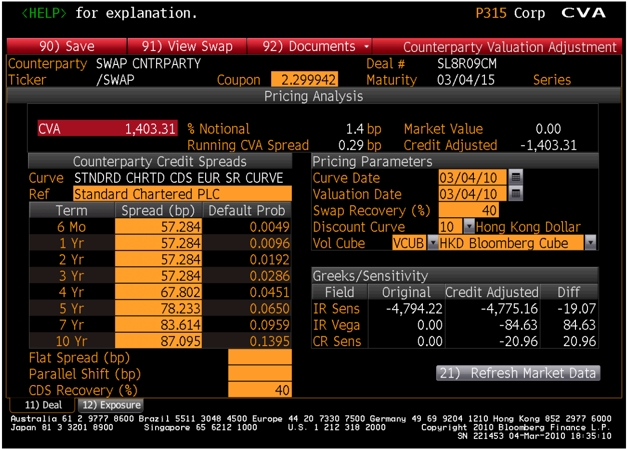
we have

$$\max(\bar{S}(t),0) = \max(S(t)+D,0) = \max(S(t),-D) + D$$

So the option to enter into the tail of the swap is the same as the option to enter into the swaption with an associated fee of $S(t) - \bar{S}(t)$ with an additional cash component of $\bar{S}(t) - S(t)$. One could apply a convexity adjustment technique to work this out, but it replacing $Z(t,t_1)$ and $Z(t,\bar{t}_1)$ with their corresponding time zero forward values is a reasonable approximation and it makes these adjustments fixed values and thus easily handled.

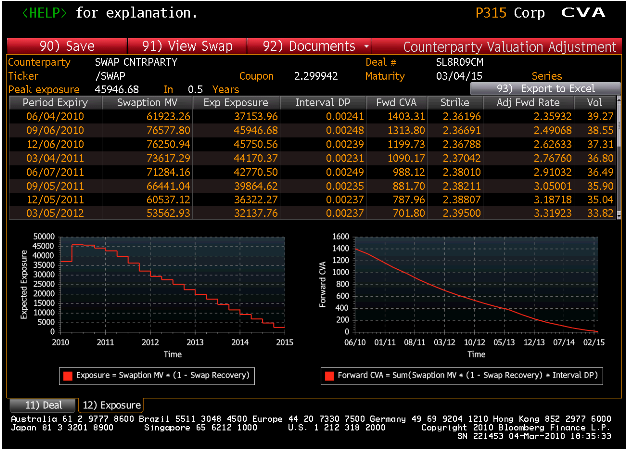
Counterparty risk in swaps — CVA<Go>

On the Bloomberg, the CVA function does this calculation.



Visualizing counterparty risk

The calculation and risks can be better visualized by exposure graphs over time:



Period Expiry	Swaption MV	Exp Exposure	Interval DP	Fwd CVA	Strike	Adj Fwd Rate	Vol
06/04/2010	61923.26	37153.96	0.00241	1403.31	2.36196	2.35932	39.27
09/06/2010	76577.80	45946.68	0.00248	1313.80	2.36691	2.49068	38.55
12/06/2010	76250.94	45750.56	0.00239	1199.73	2.36788	2.62633	37.31
03/04/2011	73617.29	44170.37	0.00231	1090.17	2.37042	2.76760	36.80
06/07/2011	71284.16	42770.50	0.00249	988.12	2.38010	2.91032	36.49
09/05/2011	66441.04	39864.62	0.00235	881.70	2.38211	3.05001	35.90
12/05/2011	60537.12	36322.27	0.00237	787.96	2.38807	3.18718	35.04
03/05/2012	53562.93	32137.76	0.00237	701.80	2.39500	3.31923	33.82

CCDS

Contingent credit default swaps are a variant of credit default swaps.

With both CDS and CCDS, the buyer of insurance pays the spread until default or maturity (whichever comes first), and receives $(1 - R)$ of *something* at default time.

What makes the two contracts substantially different is what exactly is being made whole.

- CDS — Receives $(1 - R)$ on the *principal* of a reference secrity.
- CCDS — Receives $(1 - R)$ on the *value* of a reference security.

With a CDS contract, the only uncertainty regarding the former's payout given default is the recovery rate, because recovery is on a fixed principal amount.

With a CCDS contract, there is uncertainty on the value on which recovery is applied as well.

CCDS and CVA

Consider a CCDS contract on an interest rate swap.

- The payoff of the floating leg of the CCDS contract is the loss given default for the interest rate swap.
- The value of the floating leg is then the CVA value.
- The fixed leg of the CCDS contract must then be the annuitized (to default time) value of the CVA.

The CCDS contract is thus the perfect hedge for the counterparty exposure on the interest rate swap, except that

- CCDS contracts are thought to be expensive forms in which to buy this protection.
- CCDS contracts have counterparty risk too!

Any hedge that would be used for the counterparty risk in a swap can be adjusted to serve as the replicating strategy for the CCDS contract (except for the counterparty exposure of the CCDS).

Hedging CVA

Hedging counterparty risk can be difficult. Since

$$\mathrm{CVA} \approx (1 - R) \sum S(\bar{t}_i) \bar{P}(t_i)$$

It's natural to hedge with a portfolio of swaptions and CDSs:

- Take (1-R)$\bar{P}(t_i)$ positions in swaptions maturing at $\bar{t}_i$

and

- Take positions in CDS that neutralize exposure to credit moves.

Hedging issues

By virtue of the swaption positions (and the low sensitivity of CDS to interest rates:

- Changes in CVA due to changes in interest rates and interest rate volatility are hedged.
- Changes in CVA due to changes in credit spreads are hedged.

However

- CDS position needs rebalancing when interest rates and vols change — Dynamic hedging with CDS is expensive.
- Swaption position needed changes when CDS spreads change — Dynamic hedging with swaptions is expensive.
- Cross gamma risk.
- Where can you get a risk free swap?
- What about the CVA of the CDS and the swaptions for that matter?
- How do you neutralize credit exposure and default exposure at the same time?

No CDS CDS

If CDS are not available for a given name, there are alternative ways to estimate default probabilities:

- Back out a CDS spread from a bond par curve.
- Use a generic spread (CDS indices).
- Use CDS spreads from a similar name.
- Use CDS spreads from a similar sector and credit rating.
- Use historically estimated default probabilities for a similar sector and credit rating.
- Estimate credit risks from low strike puts.

Each of these methods has its own problems.

Bond spread method

Backing out spreads from bonds is closest to yielding the price of a hedge. Problems include:

- Liquidity risk is priced into bond yields.
- Issued bonds might trade rarely or not trade at all.
- To actualize the hedge, one would have to short the bond.

Proxy CDS method

Using other CDS spreads includes:

- Use a generic spread (CDS indices).
- Use CDS spreads from a similar name.
- Use CDS spreads from a similar sector and credit rating.

On the plus side:

- Can give reasonable estimates of CDS spreads.
- Can be highly correlated with a particular name.

On the minus side:

- They all break down when that name deteriorates.
- They fail to capture idiosyncratic changes.
- They fail to capture the actual default event.
- In the latter case, a name will deteriorate well before its credit rating changes, leading to an underestimate of credit risk.

Historical default probabilities

Using historical default probabilities has its own problems.

- Same problems as the sector/credit rating approach.
- Cannot even hedge market moves.
- Does not respond to changing market conditions.
- Not a market price (historical, not implied).

Option estimation approach

Using low strike puts to estimate and hedge credit risk is an interesting approach.

- Options can be purchased to effect the hedge.
- Options capture the default event.
- Good correlation to credit risk.

On the other hand:

- Can be a disconnect between stock prices and credit (GM and Kerkorian in 2005).
- Adds substantial model risk — model must capture equity moves along with credit risk and default events.
- Too volatile — adds exposure to spot price moves.

General proxy issues

More generally, if a proxy is used which cannot effect a hedge, then

- The price is not an arbitrage free price.
- The credit risk cannot be hedged.

In this situation, one should consider managing the risk differently:

- Collateralize.
- Capitalize based on real world potential future exposures.

Portfolio counterparty risk

In the presence of netting agreements, the counterparty exposure must be computed on the portfolio of securities covered by the netting agreement.

- Loss given default is no longer the sum of the losses in the individual positions.
- Loss given default is $(1 - R)$ of the payoff of a call on the underlying *portfolio*.

Calculating portfolio counterparty risk

Calculation options:

- Feed call options on the portfolio to your defaultable interest rate derivatives valuation system.
- Compute value of appropriate call options on the portfolio via your interest rate derivatives valuation system, and proceed as above.
- Make some assumptions and get formulas.

Calculating portfolio counterparty risk

Brigo and Massimo take the latter approach.

- Net all of the interest rate swaps covered by a given netting agreement.
- The floating "leg" of the portfolio is now some sort of amortizing floating leg with time dependent leverage.
- The fixed "leg" is now some sort of amortizing step coupon fixed leg.

Difficulties ensue because the leverage could be positive or negative (i.e. - at some times the aggregate can be a payer swap while at other times it can be a receiver swap).

Portfolio woes

Consider a portfolio consisting of an at the money 5 year payer swap with fixed rate F_1, and an at the money 3 year receiver swap with fixed rate F_2.

- Credit exposure 1 year out is to the difference between the value of the two tails being positive.
- Similar to the difference between the 4 year swap rate and the 2 year swap rate.

The portfolio behaves roughly like a spread between two rates, so the credit exposure is like a spread option, and thus, difficult to price.

CVA & Basel

Basel II requires capital to be held for credit risk.

- Standardized approach — Weight by credit rating.
- Current exposure method — EAD = CE + PFE - Collateral. PFE is estimated as the notional times a multiplier based on asset type and maturity.
- Internal Rating Based approach (AKA Internal Model Method) — $1.4 \times$ Effective Expected Positive Exposure (EEPE = average Expected Exposure).

Basel III

Basel III expands on Basel II.

- IMM approved banks must account for credit spread vol on OTC derivatives — CVA VaR.
  - Advanced CVA charge — market risk rules of 10 day 99% VaR
  - Standardized CVA charge — uses formula which assumes 1 year 99.9%.
- Incremental Risk Charge — account for credit default and migration risks.
- Comprehensive Risk Measure.

Basel III CVA formula

Basel III stipulates a formula for the standard CVA charge. In our notation, it's

$$\mathrm{CVA} = (1 - R) \sum_i \max(0, e^{\frac{-s_{i-1} t_{i-1}}{1-R}} - e^{\frac{-s_i t_i}{(1-R)}}) \frac{S(t_{i-1}) + S(t_i)}{2}$$

where s_i is the credit spread at time t_i, and $S(t)$ is the current value of the option to enter into the underlying portfolio.

CVA VaR

CVA:

- Price embedded default risk of all securities under netting agreement with counterparty, taking collateralization into account.
- Requires pricing portfolio of options on the portfolio, which itself might contain options.
- Complicated model.
- American Monte Carlo.

CVA VaR

- Compute distribution of CVAs one year out.
- Compute best of the worst — best case loss of the worst X% losses (99.9% VaR on a 1 year horizon for credit risk, 99% and 10 day for Basel III).
- Simulate market moves, credit spread changes and defaults (under the real world measure) to the horizon date.

CVA VaR Issues

CVA VaR, while conceptually straight forward, raises a number of issues.

- A one year VaR horizon is much longer than typically computed in VaR — trading impact, hedging impact, and life cycle events all become significant.
- Data is sparse — CVA can be priced using market data (spreads, option prices, etc). CVA VaR needs real world default probabilities, recovery rates, ...
- Exploring the variation of a complicated model under perturbations of inputs we cannot estimate.

Accounting considerations

Accounting for counterparty risk is required by accounting standards FASB 157 (US) and IAS 39 (Europe).

FASB 157, Appendix B, Paragraph 5:

- B5. Risk-averse market participants generally seek compensation for bearing the uncertainty inherent in the cash flows of an asset or liability (risk premium). A fair value measurement should include a risk premium reflecting the amount market participants would demand because of the risk (uncertainty) in the cash flows.

Accounting considerations

Accounting rules are currently being updated and globalized.

Topic 820, Section 10 replaces FASB 157, but it strengthens the position that credit risk must be accounted for:

- 55-8. A fair value measurement should include a risk premium reflecting the amount market participants would demand because of the risk (uncertainty) in the cash flows. Otherwise, the measurement would not faithfully represent fair value. In some cases, determining the appropriate risk premium might be difficult. However, the degree of difficulty alone is not a sufficient basis on which to exclude a risk adjustment.

While accurate valuation of the embedded default risk is preferred, a number of alternative approaches have traditionally been accepted.

Shifty calculations

One alternative is the discount shift method. The CVA is calculated by shifting the discount curve by the credit spread (like in the bond approach).

When all of the swaptions are in the money, this is the zero volatility version of the CVA. For a 5 year 5% receiver swap, ignoring volatility can lead to errors of 15% to 20%.

Table: 5% 5 year receiver swap, 10 million notional, with market data yielding swap rate of 3.16%

CDS rate	CVA	Discount Shift
0	0	0
100	17,610	15,227
200	35,751	29,970
300	52,950	44,249
400	69,181	58,085

Shifty calculations

The errors from the shift approximation are illustrated by the results on an at the money swap.

Table: At the money 5 year receiver swap, 10 million notional, with market data

CDS rate	CVA	Discount Shift
0	0	0
100	11,852	7,348
200	22,870	14,316
300	33,108	20,923
400	42,620	27,189

In general:

- Errors grow as relevant swaptions go out of the money.
- Large errors when swaptions are close to the money (so that volatility plays a large part).
- Cannot be used for out of the money swaps — CVA is negative.
- Same typically holds for pay fixed swaps.

Shifty calculations

To address the problem of swaps that are a liability, the discount method is modified.

- Positions of positive value:
  - Treat as assets.
  - Reduce value according to counterparty risk (by discount curve shift).
- Positions of negative value:
  - Treat as liabilities.
  - Increase value (reduce liability) according to *investor's credit*!

Nicely symmetric and based on the theory that the money is owed to the counterparty and only a fraction of it will be paid if the investor defaults.

But adds problems of bilateral CVA.

Net current exposure

Another approach — the current net exposure method.

- Net current exposure = value at immediate risk of default to investor = max(current market value, 0).
- Cost of the risk of immediate default = cost of insuring against default via CDS on counterparty's default.
- CDS notional = net current exposure.
- CDS maturity = some measure of life of deal (maturity or duration).
- CVA = Cost of insurance = cost of fixed leg of CDS contract $\approx$ the cost of the CDS spread applied over the life of the CDS contract.

Net current exposure

The appeal of the net current exposure method lies largely in its ease of implementation.

- If you can value the positions and get the CDS spreads, you can easily compute the CVA.
- Easily extended to portfolios and to take netting agreements into account.
- Easily extended to take collateralization into account — reduce the current net exposure to the threshold level (the level beyond which the position must be collateralized).
- Easily extended into a type of bilateral CVA — do as above if position is positive, and do the reverse (impact on counterparty of one's own default) if negative.

Net current exposure

The negative of this approach comes from its inaccuracy. Consider the CDS position needed to insure the portfolio to maturity.

- Forward value of the portfolio is not constant.
- Improve hedge by using a portfolio of CDS so that the forward market values of the portfolio are matched.
- This CDS portfolio hedges against default risk assuming zero interest rate risk.
- Current net exposure method is a rough approximation of this with just one CDS contract.

As such, there are a number of disadvantages:

- Zero volatility approach, so similar problems to the discount curve shift approach — neglects value from interest rate volatility, which can be substantial.
- Even less accurate, in that only one CDS contract is used.
- Method tends to be unstable — value is proportional to market value, which has much higher volatility than the true CVA.

Bilateral CVA

In accounting circles, one often finds support for bilateral CVA calculations.

- Unilateral CVA — value of contract taking into account default of counterparty. What we've discussed up until now.
- Bilateral CVA — value of contract taking into account both default of counterparty and default of investor.

Bilateral CVA is a complicated calculation.

- Need to know relationship between default of counterparty and default of investor.
- Often approximated as difference in unilateral CVA of investor and counterparty.
- Approximation is true bilateral CVA assuming the probability of both parties defaulting before contract maturity is zero.
- Can lead to significant error with high default correlation.

Bilateral CVA

Bilateral CVA is often looked upon favorably.

- Reduces CVA charge.
- Liabilities behave more like bond liabilities - a drop in credit of the investor will potentially improve the balance sheet.

It also has some drawbacks.

- If investor's credit is worse than counterparties, bilateral CVA *increases* value of the derivative above the risk free value:
  - 2009 — Citigroup, \$2.5 billion.
  - 2011, Q3 — Goldman, \$450 million, J.P. Morgan Chase & Citigroup, \$1.9 billion each, and BofA, \$1.7 billion.
- All derivatives (even assets) increase in value when credit rating drops.
- Prices derivative without an associated hedge.

Because of these issues, accounting boards have been lobbied to reject bilateral CVA as an acceptable approach. We can hope that these issues will be addressed as IASB, FASB and other accounting standards boards work together on global convergence of accounting standards.

Issues in CVA computations

CVA computations are difficult for a number of reasons.

- Options on a portfolio are difficult to price and hedge
  - Weak models
  - Lack of securities that effectively hedge risks
- Data is difficult to come by and manage
  - CDS spreads
  - CSA legalese
  - Extra structure within and across portfolios
- CVA calculations are 6 orders of magnitude harder than option pricing
  - Level 1 — Pricing
  - Level 2 — Calibration
  - Level 3 — Sensitivities
  - Level 4 — Trading strategy analysis
  - Level 5 — Portfolio calculations of the above
  - Level 6 — VaR
  - Level 7 — CVA
  - Level 8 — CVA VaR

This stresses the interfaces between quants, risk management and IT.

Summary

- It's important to factor credit risk into valuations.
- Netting agreements and collateralization must be accounted for.
- The prices of risky bonds can be calculated based on CDS spreads.
- Bond calculations roughly correspond to just shifting the discount curve by an appropriate amount.
- Swap credit risk is more complicated because swaps can be assets or liabilities, depending on rates.
- For swaps, the discount curve shift is sometimes used, but it is not very accurate.
- The net current exposure method is used as well, but has similar shortcomings.
- Swap credit risk can be calculated using swaptions and CDS rates.
- The CVA is the value of the default leg in a CCDS on the underlying swap.
- Portfolio CVA calculations are not so easy...

Appendix — CDS Spreads

A CDS swap consists of two legs:

- A fixed leg which pays a constant spread until default or maturity.
- A floating leg which receives $(1 - R)$ on the notional of a reference bond (i.e. — makes whole on the principal).

R is determined by bankruptcy proceedings, and the fixed leg also pays accrued interest at default time (the portion of the next coupon that is due).

The *par CDS spread* is the spread that makes the prices of the two spreads match.

Default probabilities

Modeling the price of a CDS contract

- Interest rates and default are independent.
- The recovery rate R is currently known.

Then the par swap spread $C(t_n)$ for a CDS with payment times t_i and maturing at t_n satisfies the following relationship:

$$\sum^{n} C(t_n)\alpha(t_i)D(t_i)S(t_i) + \int C(t_n)\alpha'(s)D(s)P(s)ds$$
$$= \int (1-R)D(s)P(s)ds$$

where $D(t)$ is the risk free discount factor for time t, $P(t)$ is the default probability density function for time t, $S(t) = 1 - \int^t P(s)ds$ is the survival probability for time t, $\alpha(t_i)$ is the coverage for the ith coupon (e.g. roughly 1/2 for semiannual payments), and $\alpha'(t)$ is the accrued interest factor at time t.

Interpretation of terms

The par swap spread $C(t_n)$ satisfies:

$$\sum^{n} C(t_n)\alpha(t_i)D(t_i)S(t_i) + \int C(t_n)\alpha'(s)D(s)P(s)ds$$
$$= \int (1-R)D(s)P(s)ds$$

The summation is the value of the fixed payments, not counting the accrued interest:

- $C(t_n)\alpha(t_i)$ — the coupon paid at time t_i.
- $C(t_n)\alpha(t_i)D(t_i)$ — its discounted value.
- $S(t_i)$ — probability of getting this coupon.
- $C(t_n)\alpha(t_i)D(t_i)S(t_i)$ — value of this coupon (by virtue of independence assumption).

The second term is the value of the accrued interest.

- $C(t_n)\alpha'(s)$ — accrued interest that would be paid at time s.
- $C(t_n)\alpha'(s)D(s)$ — its present value.
- $P(s)$ — the probability of paying this amount.

Similarly, the third term is the value of the disbursement made at default time.

Solving for the coupon

Recall, the par swap spread $C(t_n)$ satisfies:

$$\sum^{n} C(t_n)\alpha(t_i)D(t_i)S(t_i) + \int C(t_n)\alpha'(s)D(s)P(s)ds$$
$$= \int (1-R)D(s)P(s)ds$$

Solving for $C(t_n)$, we see that the par spread is given by:

$$C(t_n) = \frac{\int (1-R)D(s)P(s)ds}{\sum^n \alpha(t_i)D(t_i)S(t_i) + \int \alpha'(s)D(s)P(s)ds}$$

Discrete and Continuous

If we knew the CDS spread for all times t, then we could use the above relationship to convert it to a default probability curve. Similarly, we can convert probability default curves back into CDS spread curves.

However, CDS spreads are only quoted for a handful of discrete times.

Like in curve stripping, to compute implied default probabilities, we need to make an assumption about either the default probability curve or the CDS spreads.

The common assumption:

- Default probabilities are derived from piecewise constant hazard rates.

Piecewise constant hazard rates

If par CDS spreads C_i are quoted for maturities T_i, we let λ_i be discrete hazard rates, and define $\lambda(t) = \lambda_i$ for $T_{i-1} < t \leq T_i$.

Then

$$\begin{aligned} S(t) &= e^{-\int_0^t \lambda(s)ds} \\ P(t) &= -dS/dt = S(t)\lambda(t) \end{aligned}$$

For $0 < t \leq T_1$, this yields:

$$\begin{aligned} S(t) &= e^{-\lambda_1 t} \\ P(t) &= e^{-\lambda_1 t}\lambda_1 \end{aligned}$$

Using this we can solve for λ_1 given C_1, and repeat for each λ to construct the default probability curve.

Pricing CDS swaps

Once a default probability curve is constructed as above, it can then be used to compute the price of a CDS contract with a non-par spread.

The price of a receive fixed contract with spread C is then:

$$\begin{aligned} &\sum^{n} C\alpha(t_i)D(t_i)S(t_i) \\ &+ \int C\alpha'(s)D(s)P(s)ds \\ &- \int (1-R)D(s)P(s)ds \end{aligned}$$

CDS on the CCP

To facilitate trading of CDS through a central clearing counterparty (CCCP, or more commonly CCP), CDS conventions have changed. On the CCPs,

- Quarterly maturities are traded.
- Particular spreads are traded, depending on geographic locale:
  - Standard North American Contract (SNAC) — 100 and 500 bp, with 40% recovery.
  - Standard European Contract (STEC) — 25, 100, 300, 500, 750 and 1000 bp, 40% recovery.
  - Standard Emerging Market contract (STEM) — 100 and 500 bp, with 25% recovery.

The convention is to quote a par spread for investment grade, and pay the difference between the price of the traded leg and the price of the par leg, where the prices are computed using the above model and a flat curve equal to the quoted spread. This difference is the "points up-front" payment.

For distressed credit, instead of quoting the par spread, the points up-front themselves are quoted.

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