VANNA-VOLGA DURATION MODEL

Kurt Smith

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Overview

- Background
- Theoretical framework
- Vanna-volga duration model
- Testing
- Results
- Conclusion
Background
Background

BSM

Underlier follows GBM with constant volatility; complete (can hedge and replicate with the underlier alone); universally accepted paradigm for exotic option TV but it does not reflect market traded reality (TV ≠ MV).

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Background

Vanna-volga

Practitioner-inspired heuristic model; BSM with exogenous volatility surface; accounts for the smile and the skew but not the term structure of volatility; disappointing pricing performance.
Background

Local vol

Retains ‘nice’ BSM features: simple, transparent, one-factor and complete. Endogenous volatility surface; local vol surface can be “very surprising and unintuitive” (Ayache et al., 2004), e.g., smile flattens over time.
Stochastic vol

Plausible in theory, but in practice model parameters implied by cross-sectional option prices are inconsistent with time series properties of underlier returns (Bates, 1996); endogenous surface; calibration “does not tell anything about how to hedge” (Hakala & Wystup, 2002, p. 276).
Background

Jump diff
Like stoch vol is plausible but incomplete. Improves fit to near-dated vanillas; difficult to estimate model parameters that govern the jump size distribution; hedging considerations are not integral to the model price.
Background

Universal vol

Contains local vol, stochastic vol and jump diffusion as special cases; recent trend to calibrate to American binary options as well as European vanilla options; computational burden is non-trivial; family of models with widely dispersed prices for identical inputs.
Background

What’s next?

All of the aforementioned models coexist because no single model or methodology dominates in the exotic option space; “the number of pricing models is virtually infinite” (Bakshi et al., 1997, p. 2003). How do we choose from the present and future model set?
Theoretical framework
Theoretical framework

- OTC vanilla options are marked-to-market owing to the universal acceptance by the market of BSM and the \textit{traded} volatility surface.

- OTC exotic options are marked-to-model “because market [traded] prices . . . are not readily available” (Hull and Suo, 2002, p. 298).

- OTC exotic option prices are made by price-makers in sell-side financial institutions. They are exposed to \textit{model risk} in pricing, hedging, limits, profits and even economic regulatory capital.

- “Model risk . . . is the risk arising from the use of an inadequate model” (Hull and Suo, 2002, p. 297).
Theoretical framework

Objective: minimise model risk

Must reflect how the exotic market trades in practice

Model
Theoretical framework

- Like Hull and Suo (2002, p. 299) the focus is on “the risk in models as they are used in trading rooms”.

- Unlike Hull and Suo, it is not assumed “that prices in the market are governed by a plausible multi-factor no-arbitrage model”; instead, it is assumed that prices in the market are governed by the economics of financial intermediation.
Financial institutions are profit driven. Model price revenues and traded hedge costs must be consistent for profit to reflect economic reality (cf. Heston; Bates; LM).

Prices are extremely model-dependent, but hedging in the market is relatively model-independent. Irrespective of the model used to price, exotic price-makers hedge high-order greeks with liquid, commoditised and cost effective vanilla strategies.

Therefore, the actual market traded hedging behaviour of price-makers should dictate the form of the pricing model if model risk is to be minimised.
Theoretical framework

Model inputs

Inter-model
- Model 1
- Model 2
- Model 3
- Model 4
- etc.

Intra-model
- Model 3a
- Model 3b
- Model 3c

Exotics
- Exotic Price 3.1
- Exotic Price 3.2
- Exotic Price 3.3

Model risk = $?

Schoutens et al. 200%
Schoutens et al. 13%
Model risk exists

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Theoretical framework

Model inputs → Inter-model → Intra-model → Exotics

- Model 1
- Model 2
- Model 3
- Model 4
- etc.

- Model 3a → ‘Implied’ Hedge 3.1
- Model 3b → ‘Implied’ Hedge 3.2
- Model 3c → ‘Implied’ Hedge 3.3

Unlike traded mkt hedges

“Very different hedging strategies”
Lipton (2002, p.61)
Theoretical framework

Minimise model risk.

Use the vol surface as it is traded in the market, i.e. BSM.

Relatively model-independent traded hedge.
The theoretical framework

- The economic connection between exotics and the traded volatility surface is price-makers using vanillas to hedge exotics. This is subject to traded market discipline.

- “Traders calibrate . . . daily, or even more frequently, to market data (Hull and Suo, 2002, p. 299). Recalibration is necessary because model dynamics do not match market dynamics. If they did, only one initial calibration would be necessary. Calibration is a mathematical connection that is not subject to traded market discipline.”
Theoretical framework

- Model price proximity to (exotic option) market traded prices.
- Model price consistency with the cost of (vanilla option) market traded hedges.
- The degree of model-independence is a key criterion not just a “regulative ideal” (Ayache et al., 2004).
Vanna-volga duration model
Vanna-volga duration model

- Philosophy
- Method
Duration model - philosophy

- Intermediation for profit requires (model) exotic revenues to be consistent with (market) traded hedge costs.

- Even though hedging exotics with exotics reduces model risk naturally, price-makers hedge exotics with vanillas because vanillas are the most cost effective source of greeks (liquid, commoditised, tight bid-ask spread).

- Price-makers hedge exotic books not individual exotic options. Books are “neutral in the lower moments and exposed to various risks in the higher moments” (Taleb, 1997, p. 149).
Duration model - philosophy

Option books

Taleb, p. 149.
• Exposed to high-order risks

Market risks

High-order
• Volga
• Vanna
• Term

Price of market risks

Traded
• Smile
• Skew
• Duration
Duration model - philosophy

<table>
<thead>
<tr>
<th>Vol. Risk \ Price</th>
<th>ZD Straddle&lt;sup&gt;a&lt;/sup&gt;</th>
<th>VN Fly&lt;sup&gt;a&lt;/sup&gt;</th>
<th>DN RR&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Level</th>
<th>Smile</th>
<th>Skew</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vega</td>
<td>&gt;&gt; 0</td>
<td>0</td>
<td>≈ 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volga</td>
<td>0</td>
<td>&gt;&gt; 0</td>
<td>≈ 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vanna</td>
<td>0</td>
<td>≈ 0</td>
<td>&gt;&gt; 0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> Long positions. DN RR is long Call / short Put.
Duration model - philosophy

**Term risk:**
The exotic option “can disappear prior to its maturity while the corresponding vanilla options cannot” (Lipton and McGhee, 2002, p. 82). This is given as an explanation for poor exotic pricing performance.
Duration model - philosophy

- If an exotic option terminates early then one can “unwind the hedge” (Wystup 2003, p. 3). However, this fails to account for term-dependent smiles and term-dependent skews observed in most markets.
Since the exotic “can disappear prior to its maturity while the corresponding vanilla options cannot” (Lipton and McGhee, 2002, p. 82), it is essential that the vanilla hedge maturity matches the expected term of the exotic option to eliminate the residual open risk.

Expected Stopping Time = $f\left(S, K, L, U, r_d, r_f, \sigma, t, T\right)$ for 1st gen exotics.
E.g. the maturity of the exotic is $T=1\text{yr}$, and the exotic is expected to terminate in only $t_d=3\text{wks.}$
Duration model - philosophy

Inverse vol surface

<table>
<thead>
<tr>
<th>p.u.</th>
<th>t_d</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volga</td>
<td>10.89</td>
<td>18.26</td>
</tr>
<tr>
<td>Vanna</td>
<td>0.068</td>
<td>0.225</td>
</tr>
</tbody>
</table>

Normal vol surface

<table>
<thead>
<tr>
<th>p.u.</th>
<th>t_d</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volga</td>
<td>6.20</td>
<td>18.26</td>
</tr>
<tr>
<td>Vanna</td>
<td>0.036</td>
<td>0.225</td>
</tr>
</tbody>
</table>

Vanna-volga duration model

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## Duration model - philosophy

<table>
<thead>
<tr>
<th>$\Delta \setminus T$</th>
<th>1wk</th>
<th>1m</th>
<th>2m</th>
<th>3m</th>
<th>6m</th>
<th>1yr</th>
<th>2yr</th>
<th>3yr</th>
<th>4yr</th>
<th>5yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10P</td>
<td></td>
<td>+1.60</td>
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<td></td>
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<td></td>
<td></td>
<td>+3.50</td>
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<tr>
<td>0.25P</td>
<td></td>
<td>+0.40</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>+0.25</td>
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<td>0$\Delta$</td>
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</tr>
<tr>
<td>0.25C</td>
<td></td>
<td>-3.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-5.30</td>
</tr>
<tr>
<td>0.10C</td>
<td></td>
<td>-5.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-10.60</td>
</tr>
</tbody>
</table>

- Whole-of-vol-surface calibration includes the slope of the term structure but in an opaque and non-tradable way.

- Define model parameters by one calibration over entire 50 vols.

- It does not make sense for exotic with term $\leq T$, to have prices affected by fly, RR and straddle $> T$. 

**Vanna-volga duration model**

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Duration model - philosophy

If domestic yields fall by 1%:

<table>
<thead>
<tr>
<th>Spot</th>
<th>3wk Fwd</th>
<th>1yr Fwd</th>
<th>5yr Fwd</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-4</td>
<td>-4.18</td>
<td>-4.80</td>
<td>-7.29</td>
</tr>
<tr>
<td>S+4</td>
<td>+3.72</td>
<td>+2.81</td>
<td>-1.06</td>
</tr>
</tbody>
</table>

A symmetric change in spot . . .
. . . is a relatively symmetric change in near-forward . . .
. . . and a highly asymmetric change in distant-forward
Duration model - philosophy

Est is a mechanism that slides (naturally and pragmatically) between the volatility of the forward and the volatility of the spot:

As spot approaches a barrier, est shortens and the vol of spot is more prominent.

As spot moves away from a barrier, est lengthens and the vol of forward is more prominent.
Duration model - method

- Find the expected stopping time of the exotic option.
- Choose a reference delta pillar for the vanilla hedge.
- Value volga and vanna at the expected stopping time.
- Market value of the exotic = $TV +$ vanilla overhedge cost.
Duration model - method

- Find the expected stopping time of the exotic option.

Est for 1st gen exotic options with continuously monitored barriers (e.g., binary and barrier options) are analytical functions of market traded and option contract inputs. E.g., Taleb (1997, p. 476) derives the solution for a single barrier $H > S$:

$$
E(t_H^T) = \frac{h}{\lambda} + \left( T - \frac{h}{\lambda} \right) \left( \frac{h}{\sqrt{T}} - \lambda \sqrt{T} \right)
$$

$$
= e^{2\lambda h} \left( T + \frac{h}{\lambda} \right) \left( \frac{-h}{\sqrt{T}} - \lambda \sqrt{T} \right)
$$

where:

$$
h = \frac{1}{\sigma} \ln \frac{H}{S}
$$

$$
\lambda = \left( \frac{r_d - r_f}{\sigma} \right) - \frac{\sigma}{2}
$$
Duration model - method

- Choose a reference delta pillar for the vanilla hedge.

Vega Neutral Fly

DM: -0.10 0.10
LM: -0.15 0.15
W: -0.25 0.25

Delta Neutral RR

0.10 0.10
0.15 0.15
0.25 0.25

Binary options
Duration model - method

- Choose a reference delta pillar for the vanilla hedge.

**Lipton and McGhee**

Liquid, commoditised delta pillars are $0\Delta$ straddle, $25\Delta$ and $10\Delta$. Hence, $15\Delta$ is found by interpolation which introduces model risk.

**Wystup**

$25\Delta$ cross-contaminates smile and skew effects for strongly asymmetric underliers.
Duration model - method

Wystup

$25\Delta$ cross-contaminates smile and skew effects for strongly asymmetric underliers.
Wystup

25Δ cross-contaminates smile and skew effects for strongly asymmetric underliers.

<table>
<thead>
<tr>
<th>1yr Delta</th>
<th>Volga p.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td>25Δ</td>
<td>12.01</td>
</tr>
<tr>
<td>10Δ</td>
<td>18.26</td>
</tr>
</tbody>
</table>

25Δ is offered relative to 10Δ
Duration model - method

Wystup
25Δ cross-contaminates smile and skew effects for strongly asymmetric underliers.

<table>
<thead>
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<th>1yr Delta</th>
<th>Volga p.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td>25Δ</td>
<td>12.01</td>
</tr>
<tr>
<td>Poly. 15Δ</td>
<td>17.39</td>
</tr>
<tr>
<td>Cub. 15Δ</td>
<td>20.04</td>
</tr>
<tr>
<td>10Δ</td>
<td>18.26</td>
</tr>
</tbody>
</table>
Duration model - method

- Value volga and vanna at the expected stopping time.

6m EUR DNT TV = 14.58\(\frac{3}{4}\)% @ spot 1.2260, 10\(\Delta\) smile = 0.705%, 10\(\Delta\) skew = 0.2%.

\[\text{est} = 0.19801 \rightarrow \text{Vanilla maturity} \sim 2.4\text{m}\]

\[
\text{Volga}_{\text{Price}} = \frac{\left[\text{Call}(\Delta, \sigma(\text{Fly}))+\text{Put}(-\Delta, \sigma(\text{Fly}))\right] - \left[\text{Call}(\Delta, \sigma(\text{TV}))+\text{Put}(-\Delta, \sigma(\text{TV}))\right]}{\text{Volga}_{\text{Fly}}} = 0.039235
\]

\[
\text{Vanna}_{\text{Price}} = \frac{\left[\text{Call}(\Delta, \sigma(\text{Call}))-\text{Put}(-\Delta, \sigma(\text{Put}))\right] - \left[\text{Call}(\Delta, \sigma(\text{TV}))-\text{Put}(-\Delta, \sigma(\text{TV}))\right]}{\text{Vanna}_{\text{RR}}} = 0.000044
\]
Duration model - method

- Market value of the exotic = TV + vanilla overhedge cost.

6m EUR DNT TV = 14.58¾% @ spot 1.2260, 10Δ smile = 0.705%, 10Δ skew = 0.2%.

\[ DNT_{MV} = DNT_{TV} + \text{dur} \cdot DNT_{Volga} \cdot \text{Volga\ Price} + \text{dur} \cdot DNT_{Vanna} \cdot \text{Vanna\ Price} \]

\[ \text{dur} = \frac{\text{est}}{\text{Maturity}} = \frac{0.19801}{0.50137} = 0.39494 \]

\[ \text{Volga\ Price} = 0.039235 \ $pts \]

\[ \text{Vanna\ Price} = 0.000044 \ $pts \]

\[ \therefore DNT_{MV} = 0.145875 + \frac{0.39494 \times 1.33734 \times 0.03924}{1.2260} + \frac{0.39494 \times -201.3351 \times 0.000044}{1.2260} = 0.160075 \]

\[ $pts \times \frac{1}{\text{spot}} = \%\ EUR \]

Vanna-volga duration model

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Testing
Testing

- Data
- Methodology
Testing - data

- Market traded prices sourced from the interbank exotic FX option market.

<table>
<thead>
<tr>
<th>Data</th>
<th>Duration Model</th>
<th>Jex et al.</th>
<th>Lipton &amp; McGhee</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX pairs</td>
<td>8</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Value days</td>
<td>89</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>No. of trades</td>
<td>338</td>
<td>16</td>
<td>Not reported.</td>
</tr>
<tr>
<td>Exotic class</td>
<td>OT, DNT, RKO, RKI, KO, KI</td>
<td>OT</td>
<td>DNT</td>
</tr>
<tr>
<td>Skews</td>
<td>+ &amp; -</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Vol term structures</td>
<td>Normal &amp; Inverse</td>
<td>Normal</td>
<td>Normal</td>
</tr>
</tbody>
</table>
Testing - data

- Spot FX rate variation was significant over the period of the market traded price sample.

<table>
<thead>
<tr>
<th>FX</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR</td>
<td>1.2890</td>
<td>1.1940</td>
</tr>
<tr>
<td>JPY</td>
<td>114.45</td>
<td>105.45</td>
</tr>
<tr>
<td>EUR/JPY</td>
<td>136.40</td>
<td>131.85</td>
</tr>
<tr>
<td>GBP</td>
<td>1.8970</td>
<td>1.7670</td>
</tr>
<tr>
<td>AUD</td>
<td>0.7795</td>
<td>0.6865</td>
</tr>
<tr>
<td>EUR/CHF</td>
<td>1.5695</td>
<td>1.5080</td>
</tr>
<tr>
<td>EUR/GBP</td>
<td>0.6655</td>
<td>0.6655&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>CAD</td>
<td>1.3965</td>
<td>1.2920</td>
</tr>
</tbody>
</table>

<sup>a</sup> Zero variation as only one market traded price for €/£.
Testing - methodology
Testing - methodology

Testing the duration model:

- DM vs. market traded exotic option prices.
- DM vs. benchmark model for comparative testing.
## Testing - methodology

<table>
<thead>
<tr>
<th>Price proximity</th>
<th>Hedge consistency</th>
<th>Model-independence</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vanna-volga model:</strong> Disappointing.</td>
<td>Inconsistent owing to the use of constants or empirics to scale the raw result.</td>
<td>Dependent as constants / empirics corrupt the model’s basis.</td>
</tr>
<tr>
<td><strong>Local volatility model:</strong> Ren et al.; Hull &amp; Suo find widespread and persistent usage in the traded market.</td>
<td>Consistent with the cost of a market traded hedge, but not with how the market hedges.</td>
<td>Dependent as local vols are sensitive to interpolation and extrapolation method.</td>
</tr>
<tr>
<td><strong>Universal volatility model:</strong> Wide model price dispersion for identical model inputs.</td>
<td>Different models imply “very different hedging” that is not consistent with market traded hedges.</td>
<td>Dependent as traded BSM market vols reconstituted arbitrarily as non-traded model parameters.</td>
</tr>
</tbody>
</table>
Testing - methodology

Duration model price vs. (exotic) market traded prices:

— Coarse grade:

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<table>
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— Fine grade:

<p>| | | | |</p>
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Vanna-volga duration model

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Testing - methodology

Microstructure of the exotic FX option interbank market:
Price-makers get net long-the-barrier from franchise flows, hence, they prefer to short-the-barrier interbank.

<table>
<thead>
<tr>
<th>Franchise product</th>
<th>Client posya</th>
<th>Bank barrier posy</th>
</tr>
</thead>
<tbody>
<tr>
<td>KO fwd</td>
<td>RKO((\Phi)) - KO(-(\Phi))</td>
<td>Net long</td>
</tr>
<tr>
<td>Shark fwd</td>
<td>Fwd + RKO</td>
<td>Long</td>
</tr>
<tr>
<td>Range fwd</td>
<td>Fwd + DNT</td>
<td>Long</td>
</tr>
<tr>
<td>Dbl shark fwd</td>
<td>Van((\Phi)) – RKI(-(\Phi))</td>
<td>Long</td>
</tr>
</tbody>
</table>

\(\Phi\) denotes a Call (\(\Phi\)=1) or a Put (\(\Phi\)=-1).
‘Longs’ prefer the barrier to touch to receive a payoff (e.g. OT, RKI, KI) or to cancel a liability (e.g. DNT, RKO, KO).

Hypotheses:
- Binaries: DNT Mkt > Mid. OT Mkt < Mid.
- Barriers: (R)KO Mkt > Mid. (R)KI Mkt < Mid.
## Results

- Number of times the market price trades within the model bid-ask.

<table>
<thead>
<tr>
<th></th>
<th>Exotic</th>
<th>Total</th>
<th>Dur &lt;= Mkt &lt;= Ask</th>
<th>% Total</th>
<th>Dur &lt;= Mkt &lt;= Ask</th>
<th>% Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNT</td>
<td>88</td>
<td>79</td>
<td>89.8</td>
<td>1</td>
<td>1</td>
<td>1.1</td>
</tr>
<tr>
<td>OT</td>
<td>86</td>
<td>79</td>
<td>91.9</td>
<td>34</td>
<td>34</td>
<td>39.5</td>
</tr>
<tr>
<td>RKO/RKI</td>
<td>78</td>
<td>67</td>
<td>85.9</td>
<td>40</td>
<td>40</td>
<td>51.3</td>
</tr>
<tr>
<td>KO/KI</td>
<td>86</td>
<td>74</td>
<td>86.0</td>
<td>44</td>
<td>44</td>
<td>51.2</td>
</tr>
</tbody>
</table>

The bid-ask spread is identical for the duration model and the benchmark model. Total = 338 options.
Results

- Exception size for market prices trading outside model bid-ask.

<table>
<thead>
<tr>
<th>Exotic</th>
<th>Duration Model</th>
<th>Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNT</td>
<td>0.01052</td>
<td>0.02542</td>
</tr>
<tr>
<td>OT</td>
<td>0.00715</td>
<td>0.02292</td>
</tr>
<tr>
<td>RKO/RKI</td>
<td>0.00139</td>
<td>0.00154</td>
</tr>
<tr>
<td>KO/KI</td>
<td>0.00030</td>
<td>0.00086</td>
</tr>
</tbody>
</table>

The bid-ask spread is identical for the duration model and the benchmark model.

\[
ARMSE = \sqrt{\frac{\sum_{i=1}^{n} (P_{i}^{\text{Model}} - P_{i}^{\text{Mkt}})^2}{n}}
\]

\[
\forall P_{i}^{\text{Mkt}} < P_{i}^{\text{Bid}} \rightarrow P_{i}^{\text{Model}} - P_{i}^{\text{Mkt}} = P_{i}^{\text{Bid}} - P_{i}^{\text{Mkt}}
\]

\[
\forall P_{i}^{\text{Mkt}} > P_{i}^{\text{Ask}} \rightarrow P_{i}^{\text{Model}} - P_{i}^{\text{Mkt}} = P_{i}^{\text{Ask}} - P_{i}^{\text{Mkt}}
\]
## Results

- Distance between model mid and market traded price.

<table>
<thead>
<tr>
<th>Exotic</th>
<th>Duration Model</th>
<th>Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNT</td>
<td>0.01554</td>
<td>0.04479</td>
</tr>
<tr>
<td>OT</td>
<td>0.01062</td>
<td>0.02824</td>
</tr>
<tr>
<td>RKO/RKI</td>
<td>0.00171</td>
<td>0.00183</td>
</tr>
<tr>
<td>KO/KI</td>
<td>0.00041</td>
<td>0.00096</td>
</tr>
</tbody>
</table>

\[
ARMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (P_{i}^{\text{Model}} - P_{i}^{\text{Mkt}})^2}
\]

\[
P_{i}^{\text{Model}} - P_{i}^{\text{Mkt}} = P_{i}^{\text{Mid}} - P_{i}^{\text{Mkt}}
\]
## Results

- Model price by interbank exotic FX option market microstructure.

<table>
<thead>
<tr>
<th>Exotic</th>
<th>Duration Model</th>
<th>Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mkt&lt; Mid</td>
<td>Mkt &gt; Mid</td>
</tr>
<tr>
<td>DNT</td>
<td>19</td>
<td>69</td>
</tr>
<tr>
<td>OT</td>
<td>56</td>
<td>30</td>
</tr>
<tr>
<td>RKO (RKI)</td>
<td>26 (6)</td>
<td>45 (1)</td>
</tr>
<tr>
<td>KO (KI)</td>
<td>22 (8)</td>
<td>52 (4)</td>
</tr>
</tbody>
</table>

*a*Market traded price. *b*Model mid-value.

**Hypotheses:**
- Binaries: DNT Mkt > Mid. OT Mkt < Mid.
- Barriers: (R)KO Mkt > Mid. (R)KI Mkt < Mid.
Results

- Errors accounting for the interbank market microstructure.

<table>
<thead>
<tr>
<th>Exotic</th>
<th>Duration Model</th>
<th>Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNT</td>
<td>0.29554</td>
<td>0.60910</td>
</tr>
<tr>
<td>OT</td>
<td>0.19118</td>
<td>0.76213</td>
</tr>
<tr>
<td>RKO/RKI</td>
<td>0.24493</td>
<td>0.57451</td>
</tr>
<tr>
<td>KO/KI</td>
<td>0.23047</td>
<td>0.64198</td>
</tr>
</tbody>
</table>

$$RRMSE = \sqrt{\frac{\sum_{i=1}^{n} ((P_{i}^{\text{Model}} - P_{i}^{\text{Mkt}}))/(P_{i}^{\text{Ask}} - P_{i}^{\text{Bid}}))^2}{n}}$$

Bid | Mid | Ask
---|---|---
\|\|\|\|\|\|\|

Vanna-volga duration model

21.03.2011
Conclusion
Conclusion

- By valuing term risk as well as smile and skew risks, the duration model achieves strong pricing performance and it identifies and quantifies a market traded hedge consistent with the model price and observed market behaviour. Volatility surface is used as intended, as an input into the BSM model.

- Duration model prices closely reflect not only the level of market traded prices but also the short-the-barrier bias in the microstructure of the interbank exotic FX option market. This is another strong indicator that it reflects the market mechanism.

- The duration model can be used by prudential supervisors to measure the financial economic risk in financial engineering models, which has a bearing on the sufficiency and efficiency of economic regulatory capital under Basel II.
Extensions

- Test using different data samples (e.g. different period, different market). This is challenging owing to OTC markets being difficult for non-participants to access.

- Test dynamic hedging performance along the lines of Engelmann, Fengler, Nalholm, and Schwender (2006); and An and Suo (2009). It is because of the strong pricing performance of the duration model in this research that the testing of its dynamic hedging performance is of interest.
References


Vanna-volga duration model 21.03.2011
References

- Ren, Y., Madan, D., Qian, M., 2007. Calibrating and pricing with embedded local volatility models.