Two Topics in Finance: Siamese Twins and The Macroinvestment Function by Steve Ross MIT Columbia University Seminar Monday, February 5, 2007

Copyright@2006 Steve Ross
Traditional Empirical Findings

- Efficient Markets:
  - Returns are serially uncorrelated – glass is full
  - It's difficult to make excess returns using fundamentals
- NA and Risk Neutral Pricing:
  - Arbitrages are hard to find
  - The engineering side of finance – builds bridges, sets prices
- Asset Pricing: “The data has yet to meet a theory it likes” - Fama
  - CAPM betas appear unrelated to pricing
  - Representative agent CBM’s fare very poorly
  - APT (ICAPM) betas are only weakly explanatory of pricing
- Corporate Finance:
  - Event studies largely support efficient market pricing and MM
A Sampling of My Favorite Anomalies

• Stock Markets:
  – small firm effects, P/E, momentum, calendar year effects, long run predictability, bubbles, equity premium puzzles

• Violations of the LOP:
  – MCI, Royal Dutch Shell/Shell Trading, 3COM/Palm Pilot, Citizen’s Public Utilities, internet stocks, closed end funds

• Volatility:
  – noisy prices – low $R^2$, stock market volatility/fundamental volatility, weekend and trading time vols

• Successful investors who seem to ‘beat the market’
  – hedge fund alphas
  – mutual fund performance persistence
  – Warren Buffett
The Financial Hurricane Scale
Example

- The Siberian stock market is a modest but flourishing and competitive regional market.
- Interestingly, for the past six years, with 4 exceptions, on the Siberian stock market, stocks have risen every Wednesday and fallen every Thursday.
- Furthermore, over the past six years, on all but 3 weekends in which the returns were modest, stocks opened lower on Monday than they closed on Friday.
Is it true?

How damaging is it?
More Examples

• More stocks names begin with the letter ‘x’ than with the letter ‘e’ and their market value is more than 1/26 of the market cap
• There are more than 5 planets in the solar system
Is it true?

How damaging is it?
Is it true?

- Risk Price/Earnings
- Momentum
- Small Firm Effects
- Long Run Return Predictability
- Equity Risk Premium Puzzles

How damaging is it?
The Revisionist Behavioral View

• The glass is empty
• Data doesn’t fit the established orthodox views
• The time is ripe for a Kuhn like seismic shift
“Science progresses funeral by funeral”*  

*Samuelson
Migration

- Anomalies rarely persist to foster new theories
- Typically they migrate westward on the hurricane scale as the supporting interpretation of the data erodes with replication and statistical analysis
- And typically they migrate south as they yield to neoclassical analysis
- E.G., Long run predictability and the equity premium puzzle
A Behavioral Mantra: Prices ≠ Fundamentals

- Unfortunately, ‘fundamentals’ are ambiguous and depend on some pricing theory, e.g., the ‘Internet bubble’
- An exception: Siamese twins, i.e., shares that have the same cash flows but sell for different prices
- Let’s try to ‘migrate’ this anomaly
Siamese Twins

• Royal Dutch Petroleum (RDP) and Shell Trading and Transport (STT) share in the Group company operating results 60:40
• BUT, their share prices don’t go in a 60:40 lockstep
Figure 1
Adjusted Share Price Ratio RDP/STT
Figure 2
Deviation from Parity RDP/STT

Time

Percent Deviation

Behavioral Analysis

- Irrational noise traders move the price away from parity
- Arbitrage is costly and risky; over long periods the price could deviate further from parity
Neoclassical Analysis

• A true challenge
• Like any good mystery: ‘Follow the cash’
Figure 5
RDP Dividend/Operating Income/STT
Dividend/Operating Income
Dividends Matter - MM Anew*

*Siamese Twins Successfully Separated!? 

• Volatilities:
  – \( \sigma(\text{RDP Price}/\text{STT Price}) = 12\%/\text{year} \)
  – \( \sigma(\text{RDP Dividend}/\text{STT Dividend}) = 14\%/\text{year} \)

• MM: How can dividends matter? Changing the timing of payouts doesn’t change firm value

• But, changing the level of payouts does

• If the dividends leave ‘value at infinity’ then raising them will raise value
  – Example: A firm has one million dollars of government bonds, but only pays a dividend of $1/year
\[ V_0 = L(<D_t>) \]

\[ = \sum_{1}^{\infty} L(D_t) \]

\[ = \sum_{1}^{\infty} E^* \left( \frac{1}{\prod_{1}^{t} (1 + r_{s-1})} D_t \right) \]

\[ V = \text{value} \]
\[ D = \text{dividends} \]
\[ r = \text{spot interest rate} \]
\[ L = \text{risk neutral valuation operator} \]
\[ E^* = \text{martingale expectation} \]
\[ D_t = \theta_t A_t \]

\[ = \theta_t A_0 \prod_{0}^{t-1} (1 - \theta_s) \prod_{1}^{t} (1 + x_s) \]

\( A = \) value of firm assets
\( x = \) rate of return on assets
\( \theta = \) payout rate
Firm valuation

\[ V_0 = L_0 \left\{ \sum_0^\infty D_t \right\} \]

\[ = L_0 \left\{ \sum_0^\infty \theta_t A_0 \prod_0^{t-1} (1 - \theta_s) \prod_1^t (1 + x_s) \right\} \]

\[ = A_0 \sum_0^\infty L\{\theta_t \prod_0^{t-1} (1 - \theta_s) \prod_1^t (1 + x_s)\} \]

\[ = A_0 [\theta_0 + (1 - \theta_0) L\{\theta_1 (1 + x_1)\} + (1 - \theta_0) L\{\theta_2 (1 - \theta_1)(1 + x_1)(1 + x_2)\} + \cdots ] \]
Theorem 1

If $\delta = \liminf \theta_t > 0$ a.s.,
then dividends exhaust the value.
Theorem 2

If there exists a convergent sequence, \( < \alpha_t > \), such that

\[
\sum_{0}^{\infty} \alpha_t = \omega < 1
\]

that majorizes payout rates, i.e., \( \alpha_t > \theta_t \), a.s., then the dividends do not exhaust the value and there is value at infinity.
Bubbles

• Value at infinity is a form of a ‘rational’ bubble
• Rational bubbles occur when the myopic valuation equation is solved for an arbitrary positive value at infinity which produces an arbitrary current value
• By contrast, we set the price equal to the discounted payouts and identify the remaining value as locked in the firm presumably because of imperfections in corporate control
RDP and STT

- Two twins share in the cash flows from the Group company in the ratio $\lambda/(1-\lambda)$
- But, if one or both leave value at infinity, then what matters is the ratio of the value of their dividend payouts
- Hence we can have

\[
\frac{V_{RDP}}{V_{STT}} \neq \frac{\lambda}{1-\lambda}
\]
<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>RDP Dividend yield</th>
<th>RDP Dividend yield</th>
<th>RDP Dividend yield</th>
<th>STT Dividend yield</th>
<th>STT Dividend yield</th>
<th>STT Dividend yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regressors:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.012</td>
<td>0.019</td>
<td>0.012</td>
<td>0.039</td>
<td>0.067</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>2.357</td>
<td>3.022</td>
<td>2.334</td>
<td>3.816</td>
<td>5.220</td>
<td>3.831</td>
</tr>
<tr>
<td>Lagged Dividend</td>
<td>0.776</td>
<td>0.708</td>
<td>0.776</td>
<td>0.227</td>
<td>-0.044</td>
<td>0.197</td>
</tr>
<tr>
<td></td>
<td>8.650</td>
<td>7.419</td>
<td>8.557</td>
<td>1.218</td>
<td>-0.235</td>
<td>1.022</td>
</tr>
<tr>
<td>Lagged Price</td>
<td>0.000</td>
<td></td>
<td></td>
<td>-0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-1.818</td>
<td></td>
<td></td>
<td>-3.044</td>
<td></td>
<td>-3.344</td>
</tr>
<tr>
<td>Forecast error in</td>
<td></td>
<td>-0.001</td>
<td></td>
<td></td>
<td>-0.011</td>
<td></td>
</tr>
<tr>
<td>Operating Earnings</td>
<td></td>
<td>-0.106</td>
<td></td>
<td></td>
<td></td>
<td>-0.660</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.606</td>
<td>0.624</td>
<td>0.598</td>
<td>0.016</td>
<td>0.241</td>
<td>-0.004</td>
</tr>
</tbody>
</table>

All variables are annual and are in adjusted per share values. The first rows show the coefficient values and below are the t-statistics. The number of observations is 50 for RDP and 30 for STT.
Table 1 Analysis

- RDP and STT have different dividend policies
- RDP dividend yield seems to depend only on lagged dividend yield
- STT dividend yield depends negatively on lagged STT price
- Reminiscent of the closed end fund findings
- Operating earnings are not significant – a weak measure of innovations?
### Table 2

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>RDP Price return</th>
<th>STT Price return</th>
<th>RDP return</th>
<th>STT return</th>
<th>RDP return</th>
<th>STT return</th>
<th>RDP return</th>
<th>STT return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regressors:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.001</td>
<td>-0.933</td>
<td>0.158</td>
<td>0.211</td>
<td>0.158</td>
<td>0.211</td>
<td>0.071</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>-1.590</td>
<td>-0.658</td>
<td>5.274</td>
<td>3.623</td>
<td>5.227</td>
<td>3.554</td>
<td>3.054</td>
<td>-0.051</td>
</tr>
<tr>
<td>Lagged Div</td>
<td>12.297</td>
<td>10.926</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.676</td>
<td>2.394</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged Pric</td>
<td>0.671</td>
<td>0.685</td>
<td>10.001</td>
<td>4.737</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RDP Dividend Yield</td>
<td></td>
<td></td>
<td>6.316</td>
<td>6.332</td>
<td></td>
<td></td>
<td></td>
<td>0.285</td>
</tr>
<tr>
<td>Forecast Error</td>
<td></td>
<td></td>
<td>2.813</td>
<td>2.797</td>
<td></td>
<td></td>
<td></td>
<td>1.452</td>
</tr>
<tr>
<td>STT Dividend Yield</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.465</td>
<td>7.641</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forecast Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.205</td>
<td>2.203</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operating Income</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.077</td>
<td>0.096</td>
</tr>
<tr>
<td>Forecast Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.467</td>
<td>0.371</td>
</tr>
<tr>
<td>STT return</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.584</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.119</td>
</tr>
</tbody>
</table>

R²                     | 0.975            | 0.951            | 0.126      | 0.114      | 0.111      | 0.087      | 0.778      | 0.036      

All variables are annual and are in adjusted per share values. The first rows are the coefficient values and below are the r-statistics. The number of observations for RDP is 50 and 30 for STT. For RDP the forecast errors for dividends are calculated as the difference between the current dividend and the forecast from the AR(1) lagged dividend model reported in Table 1. For STT the forecast errors are calculated as the difference between the current dividend and the forecast from the lagged price regression reported in Table 1. An AR(1) process was estimated to compute the forecast errors for operating income.
Table 2 Analysis

• Dividend innovations are insignificantly correlated – returns are strongly correlated
• Prices depend significantly on lagged dividends in the presence of lagged prices
• Given dividend yield innovations, operating earnings are irrelevant
• Returns for each company are correlated with contemporaneous dividend innovations
Table 3

<table>
<thead>
<tr>
<th>Dependent Variable: RDP return</th>
<th>STT return</th>
<th>RDP return</th>
<th>STT return</th>
<th>RDP return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regressors:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.192</td>
<td>0.229</td>
<td>0.189</td>
<td>0.205</td>
</tr>
<tr>
<td></td>
<td>4.831</td>
<td>4.968</td>
<td>5.087</td>
<td>4.089</td>
</tr>
<tr>
<td>RDP Dividend Yield</td>
<td>13.830</td>
<td>5.600</td>
<td>11.051</td>
<td>-5.450</td>
</tr>
<tr>
<td>Forecast Error</td>
<td>4.732</td>
<td>2.294</td>
<td>3.346</td>
<td>-3.234</td>
</tr>
<tr>
<td>STT Divide</td>
<td>4.009</td>
<td>2.678</td>
<td>4.839</td>
<td>-2.160</td>
</tr>
<tr>
<td>Forecast Error</td>
<td>1.739</td>
<td>1.201</td>
<td>1.605</td>
<td>-1.404</td>
</tr>
<tr>
<td>R²</td>
<td>0.063</td>
<td>0.424</td>
<td>0.183</td>
<td>0.345</td>
</tr>
</tbody>
</table>

All variables are annual and are in adjusted per share values. The first rows are the coefficient values and below are the t-statistics. The number of observations is 50 for RDP and 30 for STT. The forecast errors for dividends are calculated as in Table 2.
Table 3 Analysis

- Innovations in the RDP dividend yield significantly affect RDP and STT returns
- STT dividend yield innovations have no explanatory power for RDP returns
- RDP - STT returns is inversely dependent on RDP dividend yield innovations
- As with a closed end fund, STT must raise dividends to raise its value to compete with RDP?
Conclusion

• It is easy to jump to the conclusion that some observations are inconsistent with neoclassical theory

• The Siamese twin anomaly may be an object lesson in the danger of doing so
Volatility and the Investment Function
Keynes

- Original IS-LM: $I = I(r)$
- Lucas supply function approach: Disparate signals are aggregated in prices - asymmetric information is key
- Keynes: animal spirits – earliest behavioral economics?
Waiting to Invest
(Ingersoll & Ross, JF 1992)

• Projects are options on the cost of financing
• Don’t invest whenever NPV > 0
• Reconciliation: Projects compete with themselves delayed in time
• Higher volatility implies higher option value and longer wait to invest - maybe
• Note: No need for intrinsic optionality in projects
Effect is Model Dependent

• A competing force: $\exp(-rT)$ is convex in $r$
• Hence higher vol $\rightarrow$ greater project NPV at every $r$
• Tradeoff: higher vol implies option worth more alive, but higher NPV implies want to take on project earlier
Square Root Model

• $r =$ current short rate of interest
• $dr = \mu dt + \sigma \sqrt{r} \, dz$
• Projects: Point input – point output:
  • Invest $I$ and realize $\$1$ $T$ periods later
  • $B(r,t) =$ value of $\$1$ $t$ periods later
Square Root Model

- \( P(r, \sigma) = e^{-br} \)
- \( b = \frac{2(e^{\gamma T} - 1)}{((\gamma - \lambda)(e^{\gamma T} - 1) + 2 \gamma)} \)
- \( \lambda = \text{interest rate risk premium} \)
- \( \gamma^2 = \lambda^2 + 2\sigma^2 \)
- \( \nu = (\lambda + \gamma)/\sigma^2 \)
- Short rate at which IRR = 0: \( r^0 = -(1/b)\ln(I) \)
- Acceptance rate:
  - \( r^* = (1/b)\ln((\nu - b)/\nu I) = r^0 + (1/b)\ln((\nu - b)/\nu) < r^0 \)
Acceptance rate vs Sigma

Acceptance Rate vs Sigma

Sigma

Rate

Acceptance Rate
Instantaneous IRR
Stochastic Volatility Model

- Comparative statics is not conclusive
- Need a model in which volatility is stochastic
- Extension of the one factor model
Two Factor Square Root Model

• $dx = (\mu_x + \lambda_x)dt + \sigma_x \sqrt{x} \, dz_x$
• $dy = (\mu_y + \lambda_y)dt + \sigma_y \sqrt{y} \, dz_y$
• $\text{Cov}(dz_x, dz_y) = 0$
• $r = x + y$
• $\sigma_r^2 = \sigma_x^2 x + \sigma_y^2 y$
Pure Volatility Impact

• NPV preserving, i.e., neutral spread always induces delay in investment
• In the two dimensional model above, r and $\sigma^2$ are correlated and the change in vol is not NPV preserving
The Aggregate Investment Model

• In traditional neoclassical theory the available projects are arrayed by IRR from highest to lowest to create the production possibility frontier:
A Project Based Production Structure

- There is a distribution, \( h(q) \), of point input – point output projects indexed by acceptance level, \( f(r,\sigma) = q \)
- \( S(q) \) = density of stock of projects with level \( q \)
- \( \theta(q) \) = Poisson arrival rate of projects with level \( q \)
- Assume \( \theta'(q) < 0 \)
- \( \delta(q) \) = rate of shelf product decay,
- Shelf life = \( 1/\delta(q) \)
Acceptance Curves

Volatility Rate

\[ f(r, \text{sig}) = q \quad f(r, \text{sig}) = q' > q \]
Investment Dynamics

if $f(r, \sigma) > q$

- $dS(q) = \theta(q)dt - \delta(q)S(q)dt$

if $f(r, \sigma) \leq q$

- $dS(q) = \theta(q)dt - \delta(q)S(q)dt - S(q)$
  - Note: Could allow for a slower take down of projects that pass $q$ threshold
Steady State

if \( f(r, \sigma) > q \)

then \( S(q) \) approaches the steady state value:

\[ \frac{\theta(q)}{\delta(q)} \]
Path Dependence

• Suppose $q = f(r, \sigma)$ has been steady for a while
• Now suppose that $q$ rises from $q'$ to $q'' > q'$
• Along the rising path there is little or no stock of projects on the shelf since they have all been undertaken at the lower $q$.
• Hence, the rise in $q$ will only shut off the flow during the rise
Investment as $q$ rises from $q' \uparrow q''$

\[ I = \int_{q'} S(x)dx = \int_{q'} \theta(x)dx \]

Investment is a flow and the flow declines as $q$ rises
Path Dependence

• Suppose $q = f(r, \sigma)$ has been steady for a while

• Now suppose that $q$ falls from $q'$ to $q'' < q'$

• Along the falling path projects on the shelf are taken on

• At $q'$ the investment eventually settles down to the path independent steady state rate
Investment as $q$ falls from $q' \downarrow q''$

$$I = \int q'' S(x) \, dx = \int q'' \theta(x) \, dx$$

$$I = \int_q^{q'} S(x) \, dx = \int_q^{q'} \theta(x) \, dx$$

Along path total investment =

$$\int I \, dt = \int_q^{q'} S(x) \, dx$$ a stock amount
Vol and Rate Matter

- Notice that a lower interest rate by itself might have little or no impact on investment if its offset by a sufficient rise in the volatility of rates.
- If we move to lower $q$, though, this should result in a larger significant increase in investment than the decline in investments from a rise in $q$. 
Investment vs Acceptance Level

Acceptance Level, \( f(r, \sigma) = q \)

Investment Flow

Acceptance Level

0.9 0.95 1 1.05 1.1
Conclusion

• The traditional neoclassical development of the macro investment function is misleading.
• Investment is determined not only by the level of interest rates but also by their perceived volatility.
• Investment is path dependent.
• Policy impacts both of these factors.