Hard-To-Borrow Stocks, Volatility and Bubble Dynamics: a challenge to Jarrow + Protter?

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Certain stocks have limited floats. Because of current [misguided, in my opinion] regulations, stocks may only be shorted if the seller finds a lender.

Clearing firms act as clearing houses for supplies of long stock.

If a clearing firm cannot borrow stock to cover short holdings, then traders with short positions are subject to buy-ins, where the clearing firm acts to force a covering purchase on the trader’s account.

There are several consequences of these hard-to-borrow situations:

- How the stock trades
- How the options are priced
In normal circumstances, a clearing firm will permit you to short stock and pay you a *short* rate.

In a hard-to-borrow situation, they may reduce this rate, or even assess you a *negative* rate. Sometimes they will *forbid* a short sale (although this is waived for market-makers hedging a long delta position).

So the net result of all these factors is that puts and calls are not replacements for long and short stock in the traditional sense.

- Owning calls makes you long in a way unhedgeable with a stock sale.
- Puts are invaluable sources of short deltas.
- Even without a negative interest rate imposed by the clearing firm, put-call parity generally *implies* a negative interest rate.
The direct way to measure the degree of hard-to-borrowness in a stock is by pricing a conversion. (This means comparing the price of call vs. put + 100 shares stock)

For ordinary stocks carrying long stock entails a cost and hence one receives a credit for converting.

In hard-to-borrows this credit can become a (sometimes) substantial debit.

The degree of hard-to-borrowness is not fixed but varies with time. It is strongly tied to the short-interest in the stock (the percent of the float currently held as shorts).
When buy-ins take place, the stock, even in strongly down markets can have extreme, spiky, up days. Basically, the stock may be artificially taken up and then (around 3:40 ET, typically) purchased for the shorts. Imagine you have a delta-neutral position, long 100 calls, short 100 puts and short 10000 shares of stock. –$S_0=20.00$, at 9:30. By 3:40 $S=21.80$ and you are bought in. Although the clearing firm has said you may be bought-in on up to 6400 shares, you don’t know if or how much they have purchased for your account.

–Scenario A: do nothing  –Scenario B: sell 3000 shares
–$S_{fin}=21.00$

Suppose the eventual buy-in was 1426 shares!
• Case A: you are long 1426 deltas down 80 cents
• Case B: lucky, you come in the next day short 1574 deltas, but you made about $1250.
examples of hard-to-borrow stocks

- A few characteristics of hard-to-borrow stocks may seem odd:
  - end of the day spikes
  - strange in-the-money option pricing
  - regimes of high volatilities at high prices
  - reduced hard-to-borrowness after crashes
The following slide shows an actual buy-in in the biotech stock AGIX in early February, 2007. Look at the time of day when the buy-in occurred. This is typical of buy-ins.
Jan'09 2.50/8.00 put spread (11 months out 2/20/08).

Midpoint 'rule': worth = 1.25

Extremely hard-to-borrow: \( c_{(pop)} - p_{(pop)} = -0.45 \)

5xR(x(331/360)) = -0.45; \( R = -9.8\% \)
VMW before the Jan 2008 earnings announcement:
(note the lengths of the candlesticks before and after crash)
Before the earnings announcement, the at-the-money, Jan 2009 conversion traded for -$9.00. After it traded for -$1.80. So the hard-to-borrowness was enormously reduced by the crash. What is the consequence of this?

Suppose you owned 100 puts, 10000 shares of stock and were short 100 calls on the 90-line in Jan 2009.

Answer: you lost $72K
From 2001 to 2004, Krispy Kreme was extremely hard to borrow, with frequent buy-ins. The candlesticks show the stock was very volatile and high-priced reaching $200 (unadjusted).
in order to understand hard-to-borrow stocks one needs to acknowledge that with the market in normal equilibrium puts and calls are functionally at parity. if they were not, there would be a continuous order flow out of equilibrium, e.g. buying puts and selling calls. But in the real world, prices adjust to remove this scenario.

this functional parity induces an effective dividend rate as we shall see.
dynamics and pricing

- there are several papers in the literature analyzing the price of options in hard-to-borrows.
- none of these connect stock dynamics with pricing.
- we believe this misses a crucial point—that hard-to-borrowness is not an add-on trait, but is critically intertwined with stock dynamics. we revisit this property later by connecting up h-t-b’s with bubbles.
the model parameters

- $S$, stock price
- $\lambda$, buy-in frequency ($\lambda_0$, long-term)
- $\gamma$, jump size caused by buy-in
- $\sigma_0$, baseline vol of stock
- $\beta$, “coupling” of stock price to short-interest
- $\kappa$, $\alpha$, parameters delimiting the short-interest process
the model

- \( dS/S = \sigma_0 d\Omega + \gamma \lambda dt - \gamma dN_t^\lambda \)
- \( d\lambda/\lambda = \kappa dZ + \alpha (\log \lambda_0 - \log \lambda) dt + \beta dS/S \)

- some comments:
  - as buy-ins pump the stock price, they elicit more buy-ins
  - for option pricing, the short stock holder does not collect the jump
  - suggestive of, but different from, Volterra prey-predator
  - for simplicity, the short interest is embedded in the buy-in rate by the assumption of direct proportionality
  - again for simplicity, the entire jump is lost to the short holder; the reality is that not all shorts are subject to each buy-in and the amount of the buy-in is often a fraction (less than 1) of the entire short
aside (sometimes there is a full retracement!)
a speedster’s solution to the model

- **up move:** $\gamma \lambda dt$  \hspace{1cm} prob: $1-\lambda dt$
- **down move:** $\gamma \lambda dt - \gamma$  \hspace{1cm} prob: $\lambda dt$
- $\mathbb{E}(R) = 0$
- $\mathbb{E}(R^2) = -\gamma^2 \lambda^2 (dt)^2 (1-\lambda dt) + \gamma^2 (1-\lambda dt)^2 \lambda dt$
- $\mathbb{E}(R^2) = \gamma^2 \lambda dt$

**Hence:** $\sigma_{\text{eff}}^2 = \sigma_0^2 + \gamma^2 \lambda$

*(model: diffusion limit)*
a speedster’s solution to the model (2)

○ substituting the last into the expression for $\lambda$:

$$\frac{d\lambda}{\lambda} = \kappa dZ + \beta \sigma_0 d\Omega + \beta \gamma (\lambda dt - dN^\lambda_t)$$

$$\frac{d\lambda}{\lambda} = \sqrt{\kappa^2 + (\beta \sigma_0)^2 + (\beta \gamma)^2 \lambda} \ dW$$

$$+ \alpha (\log \lambda_0 - \log \lambda) dt$$
a speedster’s solution to the model (3): option pricing

○ recall for a short stock position the jump is lost
○ \[dC = C_S dS + C_t dt + C_\lambda d\lambda + \ldots\]
○ financing long call + short stock:
  \[\text{P/L} = dC - \Delta dS \quad \text{if no jump:} \quad (1-\lambda dt)\]
  \[= dC \quad \text{if jump:} \quad (\lambda dt)\]
○ \[C[S_{t+\Delta t}, \lambda_{t+\Delta t}, t+\Delta t] - C[S_t, \lambda_t, t] = -\Delta[S_{t+\Delta t} - S_t]\] no jump
○ \[C[S_t - \gamma, \lambda, t+\Delta t] - C[S_t, \lambda, t] = SC_{S} (-\gamma) (\lambda dt)\]

○ the latter term has no BS analogue
○ Hence: \[d_{\text{eff}} = \gamma \lambda\] is an effective dividend rate
a speedster’s solution to the model (4): volatility

- recall: $\sigma_{\text{eff}}^2 = \sigma_0^2 + \gamma^2 \lambda$
  $\sigma_{\text{eff}}^2 = \sigma_0^2 + \gamma \ d_{\text{eff}}$

- what you see:
  $\sigma^2 = \langle 1/T \int_0^T \sigma_{\text{eff}}^2(s) ds \rangle$
  $= \sigma_0^2 + \gamma \ 1/T \int_0^T d_{\text{eff}} \ dt$
  $= \sigma_0^2 + \gamma \ < d_{\text{eff}} >$
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American options

- In the previous slide one can see that the effective dividend rate is fairly flat, but has a broad systematic increase to higher strikes and a sharper drop off to low strikes.
- This is a consequence of the *early exercise* condition on puts and calls.
- That calls become an early exercise as a result of the dividend rate is better understood financially:
  - There is a penalty for holding short stock and a deep call: the buy-in
  - When this penalty exceeds the price of a put on the same strike, the short holder must exercise his call
  - **This is the direct reason for the fat DNDN spread**
- Put early exercise works differently: a deep-in-the-money put can be replaced with a less deep put reducing the c.c.- hence the broadly increasing $d_{\text{calc}}$. 
The following two slides show a generated dynamics for low and high frequency buy-in rates. The thing to notice is the *pumping* of the stock as buy-ins drive the stock higher and lead to further buy-ins. Our model exhibits *bursting*, or *intermittency*, buy-ins lead to more buy-ins which in turn pump up the stock and leave it in a high volatility state. At other times the stock is in a more quiescent state, low buy-in rate, low effective volatility.
Initial Lambda=2, effective dividend=1%, Gamma=0.05

Bursting behavior due to sporadic increase in buy-in rate.
Initial Lambda=20, effective dividend=10%, Gamma=0.05

Kappa=1%, Beta=5, alpha (mean rev for BIR)=1yr
KKD, VMW revisited

- A cursory scan of the previous few slides indicates that drops in hard-to-borrowness frequently attend sudden stock drops (VMW).
- Long periods of extreme hard-to-borrowness can be followed by long periods of quiet low volatility (KKD).
bubbles

- there has been a tendency to invoke the term bubble to describe exactly the kind of phenomenon we have been describing here.
- by this, we mean *perceived* high prices combined with *perceived* high volatility, punctuated by dramatic crashes.
anatomy of a bubble

early

late

(same authors)
the generic approach to bubbles has been to assert that a **valuation measure** of a sort exists which allows the astute analyst to determine that prices are **out-of-equilibrium**.

- this has the advantage by analogy with other non-equilibrium systems in that the duration of the out-of-equilibrium state may be difficult to predict; prices may be difficult to reconcile; temporal behavior (dynamics) may be weakly- or un-predictable.
we assert something radically different: hard-to-borrow stocks are a manifestation of one type of bubble [most of the 90’s internet stocks were h-t-b].

we describe them in an (extended) equilibrium fashion.

evidence for this approach lies in the consistent, persistent and actual pricing of equity options, and the intermittency of h-t-b regimes in real stocks.
we make a direct connection to underlying properties of the stocks, short-interest/buy-in rate, so our model is:

- testable by historical experiment
- predictive of intermittency rates

by analogy with physics, one can ask if h-t-b’s are a stand-alone example or characteristic of a broad universality class of bubbles, and thus a prototype for fruitful exploration.
The political conclusions are these:

to avoid bubbles and the frenetic volatility which accompanies them, reduce restrictions on short sales!
three chinese stocks
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