Attilio Meucci

Managing Diversification

Common Measures of Diversification

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}$$

portfolio return

returns of securities
(stocks, bonds, options,
structured products, …)

portfolio weights

Common Measures of Diversification

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$

weight-based definitions

$$\mathcal{D}_{Her} \equiv 1 - \mathbf{w}'\mathbf{w}.$$

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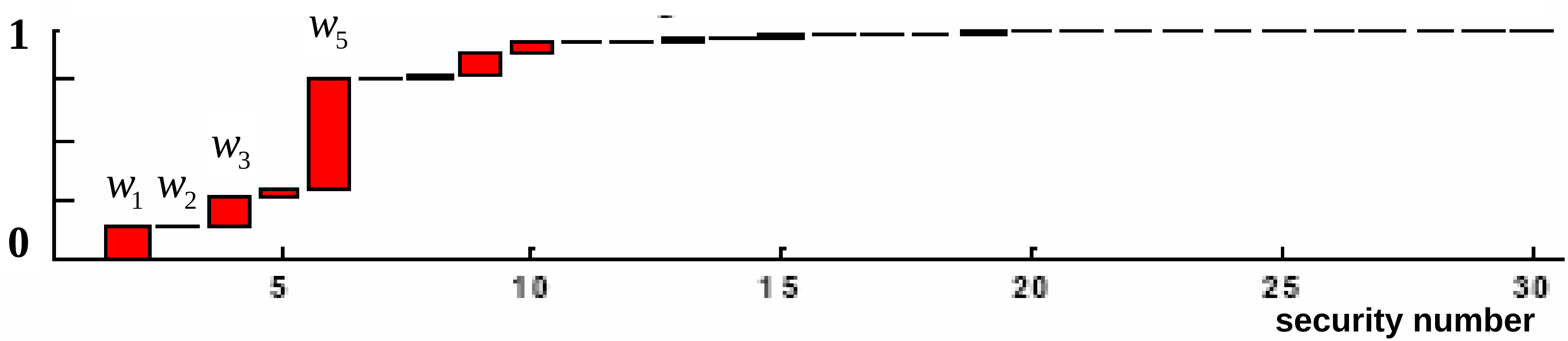
distribution

⇕

portfolio weights

- positive

- sum to one



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$$\mathcal{D}_{Her} \equiv 1 - \mathbf{w}'\mathbf{w}.$$

$$\mathcal{D}_{BP} \equiv -\sum_{n=1}^{N} w_n \ln(w_n)$$ **entropy**

$\Leftarrow$ **distribution**

$\Updownarrow$

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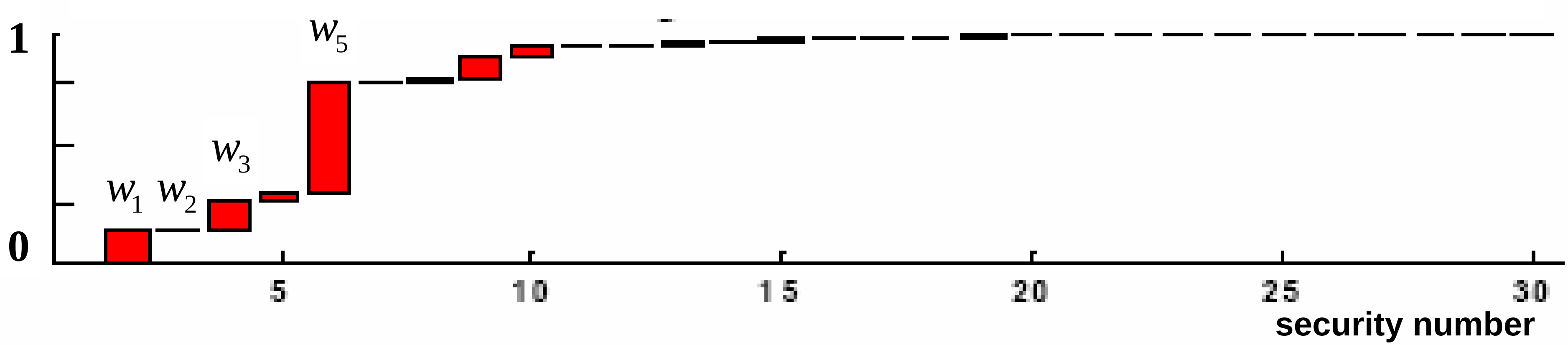

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$$\mathcal{D}_{HK}^{(\gamma)} \equiv -\left(\sum_{n=1}^{N} w_n^{\gamma}\right)^{\frac{1}{\gamma-1}}$$

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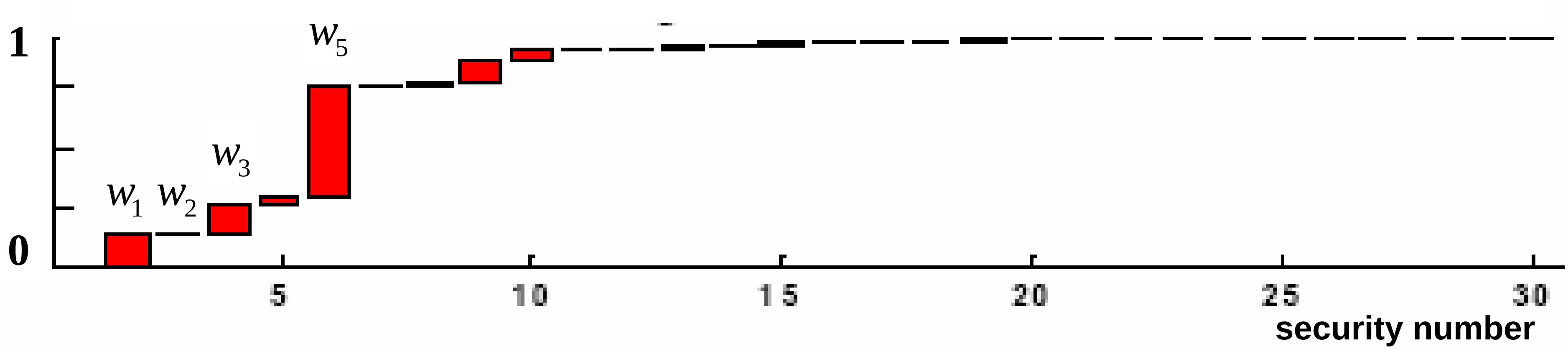

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risk-based definitions

returns correlation matrix

$$\mathcal{D}_{IP} \equiv 1 - \mathbf{w}'\mathbf{C}\mathbf{w},$$

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risk-based definitions

$$\mathcal{D}_{IP} \equiv 1 - \mathbf{w}'\mathbf{C}\mathbf{w},$$

returns standard deviations

$$\mathcal{D}_{Dif} \equiv \boldsymbol{\sigma}'\mathbf{w} - \sqrt{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}$$

returns covariance matrix

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factor-based definition

$$R_n \equiv \sum_{k=1}^{K} \beta_{n,k} F_k + \epsilon_n$$

$$\mathcal{D}_{IS} \equiv 1 - \frac{\mathrm{Var}\{R_{\epsilon}\}}{\mathrm{Var}\{R_{\mathbf{w}}\}}$$

portfolio return due to “idiosyncratic”

risk-based definitions

$$\mathcal{D}_{IP} \equiv 1 - \mathbf{w}'\mathbf{C}\mathbf{w},$$

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These definitions apply in specific circumstances and or under restrictive hypotheses

Diversification Distribution

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$

if correlations = 0 $R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}$

<u>Example</u>: portfolio of two securities

- **one bond** $w_1 = 50\%$ $\mathrm{Var}\{R_1\} = (1\%)^2$
- **one stock** $w_2 = 50\%$ $\mathrm{Var}\{R_2\} = (30\%)^2$

Diversification Distribution

if correlations = 0

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}$$

$$\boxed{\mathrm{Var}\{R_{\mathbf{w}}\} \equiv \sum_{n=1}^{N} \mathrm{Var}\{w_n R_n\}}$$

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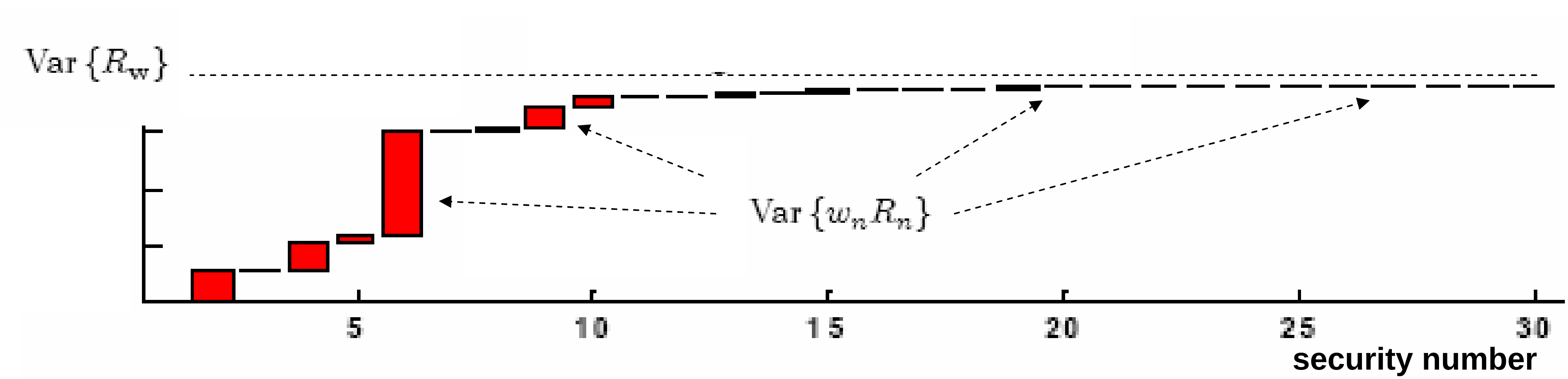
weighs highly diversified

risk highly concentrated



Diversification Distribution

if correlations ≠ 0

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$

$$\boxed{\operatorname{Var}\{R_{\mathbf{w}}\} \not\equiv \sum_{n=1}^{N} \operatorname{Var}\{w_n R_n\}}$$

Diversification Distribution

if correlations = 0

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$

$$\boxed{\mathrm{Var}\{R_{\mathbf{w}}\} \not\equiv \sum_{n=1}^{N} \mathrm{Var}\{w_n R_n\}}$$

<u>Example</u>: portfolio of two government bonds in same duration bucket

Bond 1 $w_1 = 50\%$ $\mathrm{Var}\{R_1\} = (1\%)^2$

Bond 2 $w_2 = 50\%$ $\mathrm{Var}\{R_2\} = (1\%)^2$

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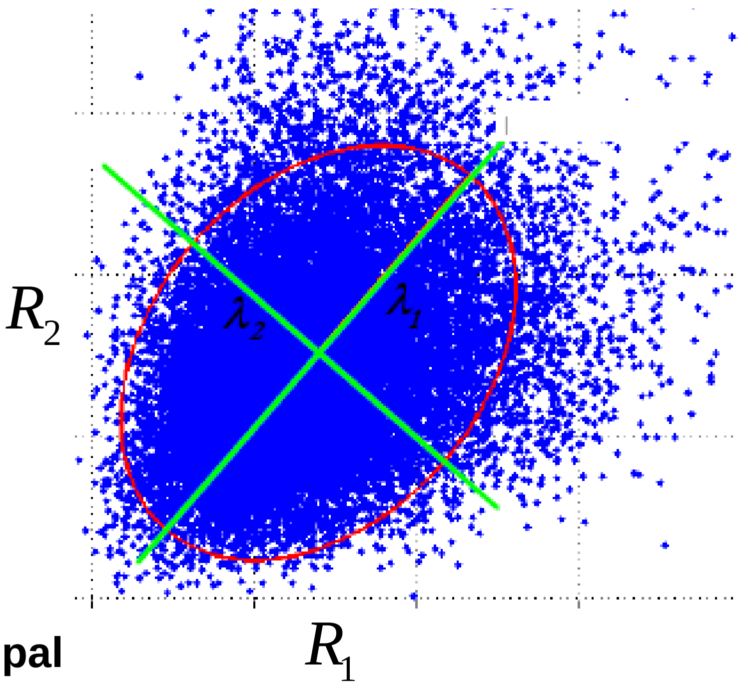
weighs highly diversified

volatility homegeneous

high concentration due to correlations: full exposure to first principal component

Diversification Distribution



$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}$$

$$\Sigma \equiv \mathrm{Cov}\{\mathbf{R}\}$$

$$\downarrow$$

$$\mathbf{\Sigma} \equiv \mathbf{E}\,\mathbf{\Lambda}\,\mathbf{E}'$$

PCA

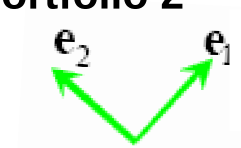
$$\mathbf{E} \equiv (\mathbf{e}_1, \ldots, \mathbf{e}_N)$$

eigenvectors

$\Updownarrow$

principal portfolios

principal portfolio 2



principal portfolio 1

$$\mathbf{\Lambda} \equiv \mathrm{diag}\left(\lambda_1^2, \ldots, \lambda_N^2\right)$$

eigenvalues

$\Updownarrow$

principal variances

$$\lambda_n^2 \equiv \mathrm{Var}\{\mathbf{e}_n'\mathbf{R}\}$$

Diversification Distribution

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$\widetilde{\mathbf{R}} \equiv \mathbf{E}^{-1}\mathbf{R}$ return of principal portfolios

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$$\left.\begin{array}{ll} \widetilde{\mathbf{R}} \equiv \mathbf{E}^{-1}\mathbf{R} & \text{return of principal portfolios} \\ \widetilde{\mathbf{w}} \equiv \mathbf{E}^{-1}\mathbf{w}. & \text{weights of original portfolio on principal portfolios} \end{array}\right\} \quad R_{\mathbf{w}} \equiv \widetilde{\mathbf{w}}'\widetilde{\mathbf{R}}.$$

A. MEUCCI - Managing Diversification Diversification Distribution

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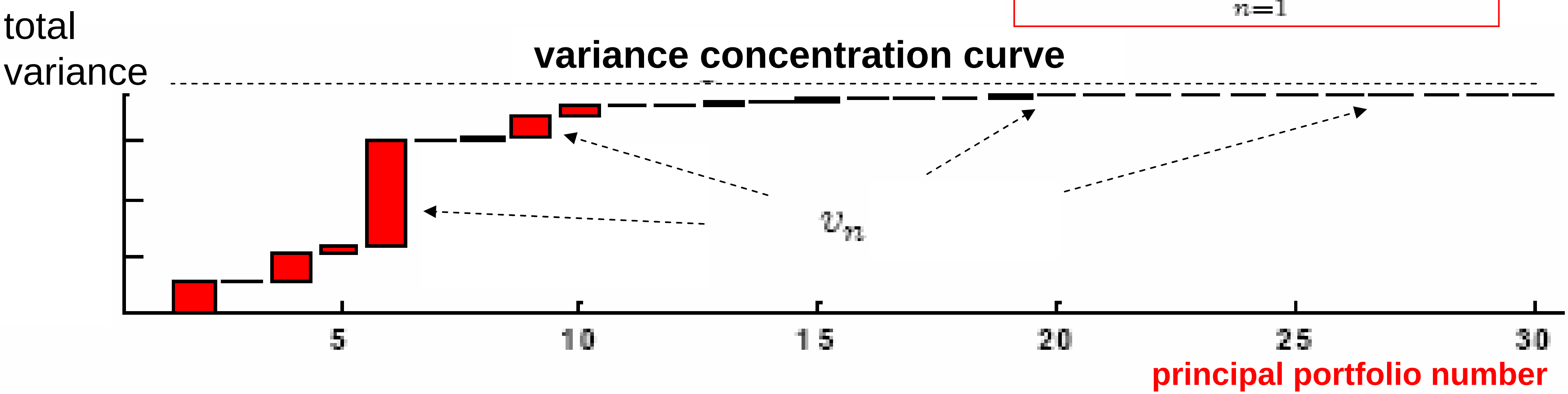
$$\boxed{\mathrm{Var}\{R_{\mathbf{w}}\} \not\equiv \sum_{n=1}^{N} \mathrm{Var}\{w_n R_n\}}$$



$$\widetilde{\mathbf{R}} \equiv \mathbf{E}^{-1}\mathbf{R}$$ return of principal portfolios

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$$R_{\mathbf{w}} \equiv \widetilde{\mathbf{w}}'\widetilde{\mathbf{R}}.$$

$$v_n \equiv \widetilde{w}_n^2 \lambda_n^2.$$ variance concentration curve

contribution to original portfolio variance from n-th principal portfolio:

$$\boxed{\mathrm{Var}\{R_{\mathbf{w}}\} \equiv \sum_{n=1}^{N} v_n}$$

Diversification Distribution

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weighs highly diversified

volatility homegeneous

variance concentration curve loads on one principal portfolio

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Diversification Distribution

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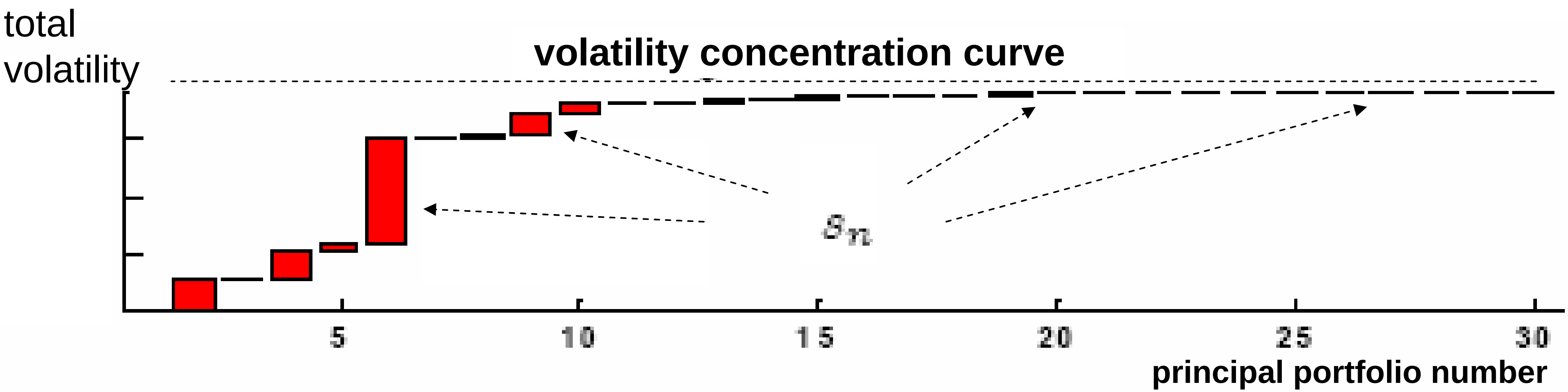

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$v_n \equiv \widetilde{w}_n^2 \lambda_n^2.$ variance concentration curve

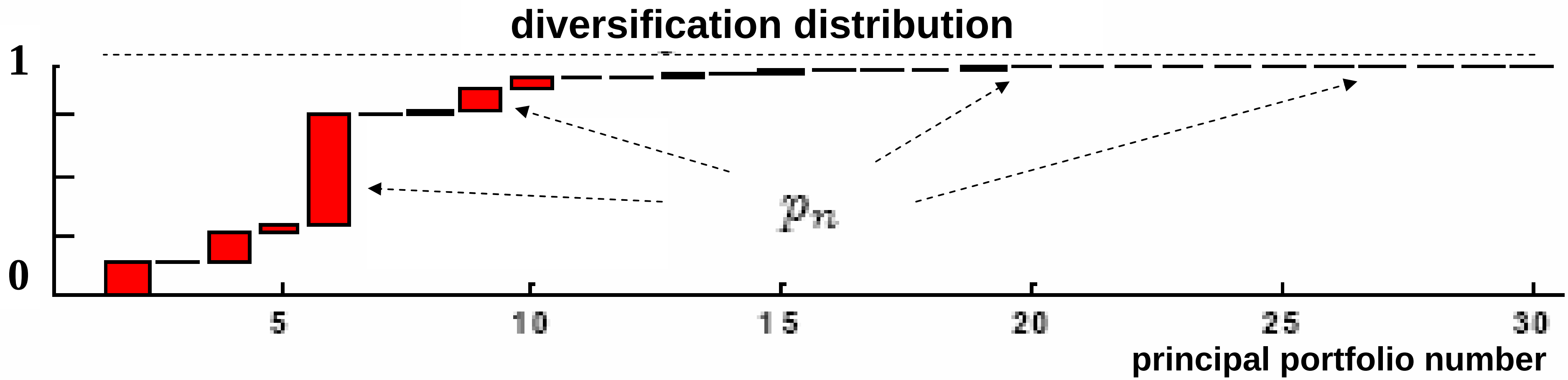
$\Updownarrow$

$s_n \equiv \dfrac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Sd}\{R_{\mathbf{w}}\}}$ volatility concentration curve

contribution to original portfolio volatility from n-th principal portfolio: "hot spots"

Diversification Distribution

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}$$



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$\Updownarrow$

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$\Updownarrow$

$p_n \equiv \dfrac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Var}\{R_{\mathbf{w}}\}}$ diversification distribution

contribution to original portfolio r-square from n-th principal portfolio

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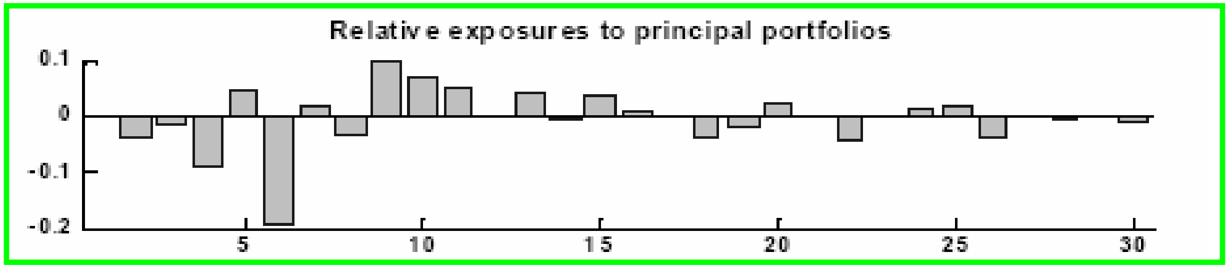
Example:
management with
benchmark



$$\mathbf{w} \mapsto \mathbf{w} - \mathbf{b}$$

portfolios weights   benchmark weights

relative weights

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$$v_n \equiv \tilde{w}_n^2 \lambda_n^2.$$ variance concentration curve

$\Updownarrow$

$$s_n \equiv \frac{\tilde{w}_n^2 \lambda_n^2}{\mathrm{Sd}\{R_{\mathbf{w}}\}}$$ volatility / **tracking error** concentration curve

$\Updownarrow$

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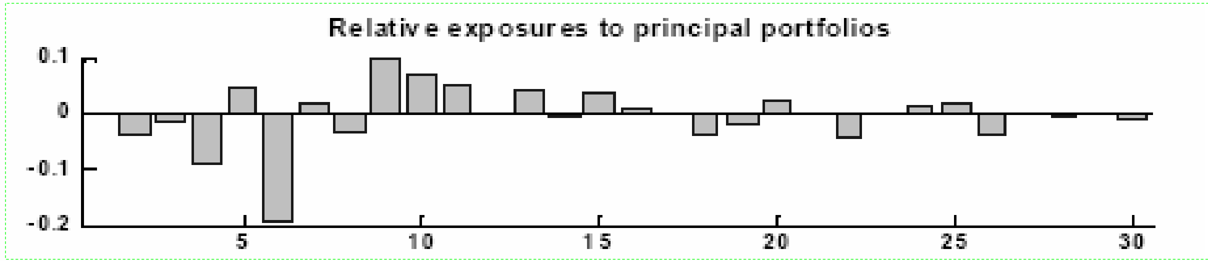
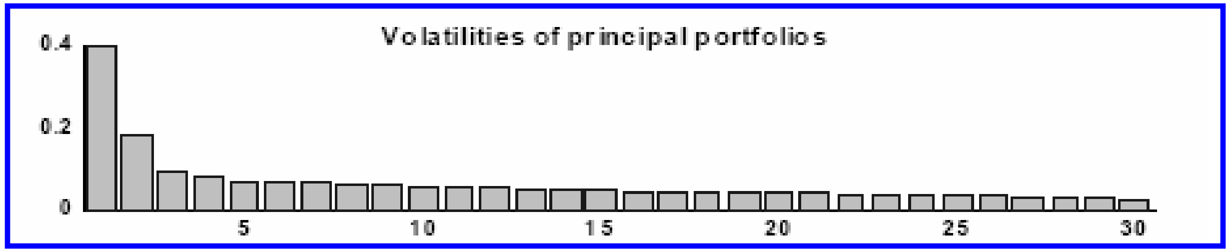
Example: **management with** **benchmark**

$$\mathbf{w} \mapsto \underbrace{\mathbf{w} - \mathbf{b}}_{\text{relative weights}}$$





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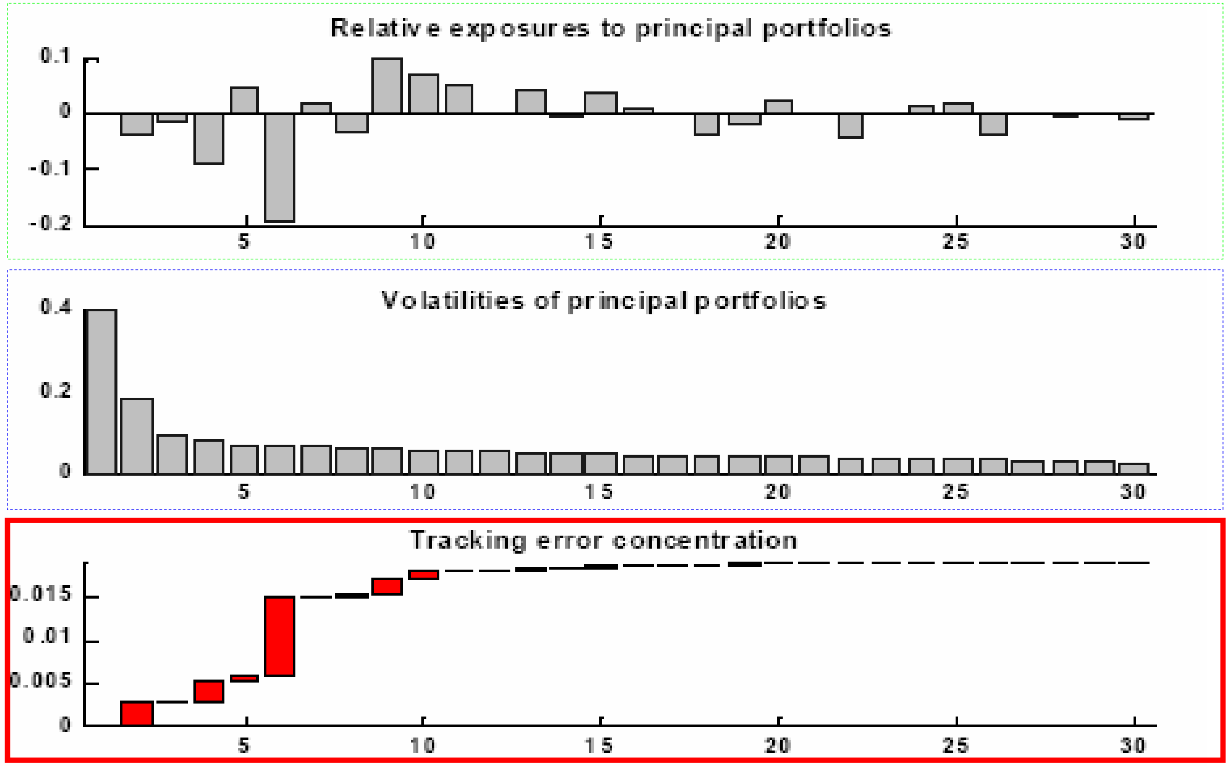
$$\begin{cases} v_n \equiv \widetilde{w}_n^2 \boxed{\lambda_n^2} & \text{variance concentration curve} \\ & \Updownarrow \\ s_n \equiv \dfrac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Sd}\{R_{\mathbf{w}}\}} & \text{volatility / tracking error concentration curve} \\ & \Updownarrow \\ p_n \equiv \dfrac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Var}\{R_{\mathbf{w}}\}} & \text{diversification distribution} \end{cases}$$

Example:
management with **benchmark**

$$\mathbf{w} \mapsto \underbrace{\mathbf{w} - \mathbf{b}}_{\text{relative weights}}$$



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COMMON MEASURES OF DIVERSIFICATION

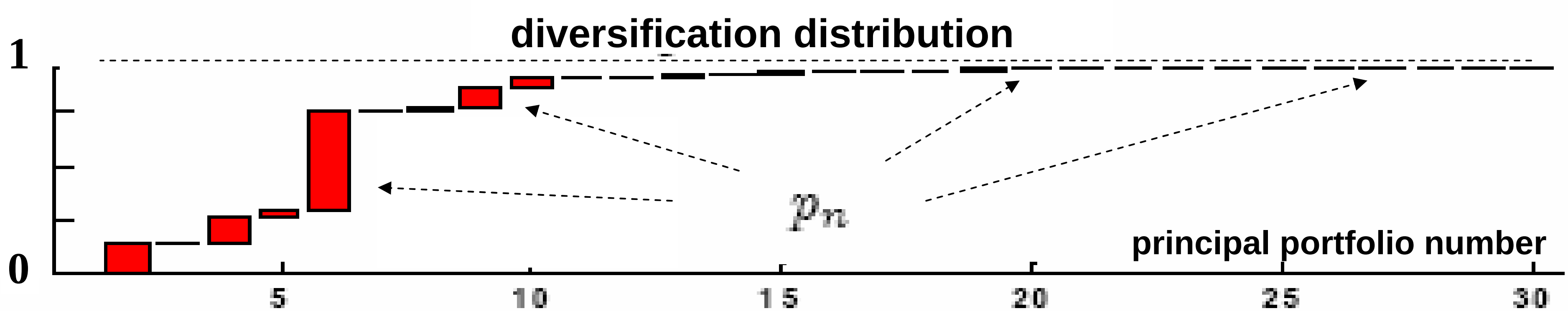
DIVERSIFICATION DISTRIBUTION

$\boxed{\text{MEAN-DIVERSIFICATION FRONTIER}}$

CONDITIONAL ANALYSIS

REFERENCES

Mean-Diversification Frontier



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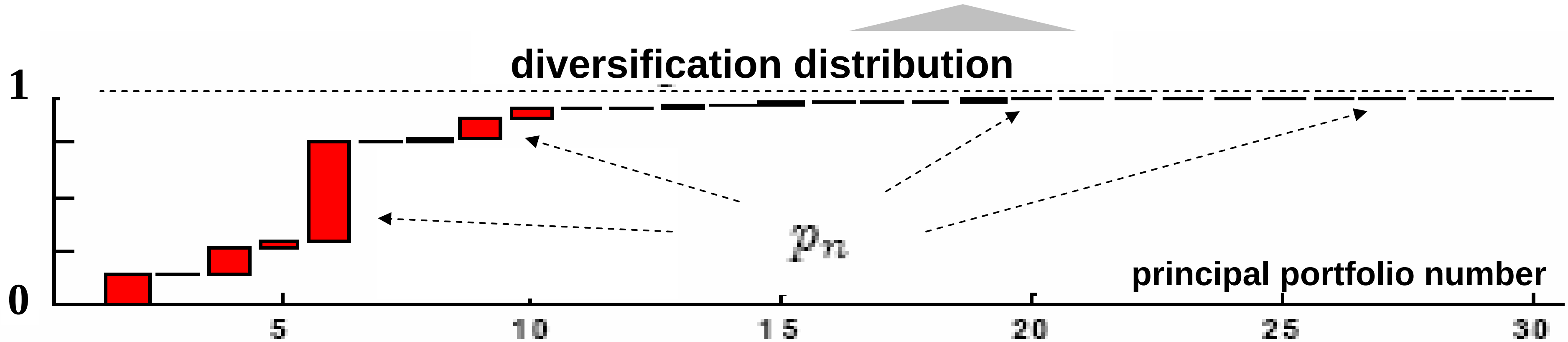
$$\boxed{p_n \equiv \frac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Var}\{R_{\mathbf{w}}\}} \quad \text{diversification distribution: “probability mass”}}$$

Mean-Diversification Frontier

diversification index ?



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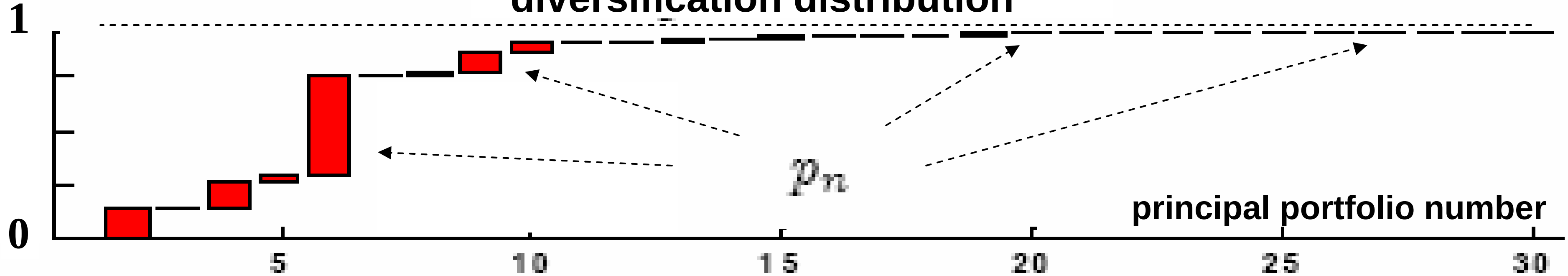
$p_n \equiv \dfrac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Var}\{R_{\mathbf{w}}\}}$ diversification distribution: "probability mass"

Mean-Diversification Frontier

diversification index → **entropy**

$$-\sum_{n=1}^{N} p_n \ln p_n$$

diversification distribution



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Mean-Diversification Frontier

Effective number of bets

$$\mathcal{N}_{Ent} \equiv \exp\left(-\sum_{n=1}^{N} p_n \ln p_n\right)$$

diversification index

entropy

$$-\sum_{n=1}^{N} p_n \ln p_n$$

Mean-Diversification Frontier

Effective number of bets

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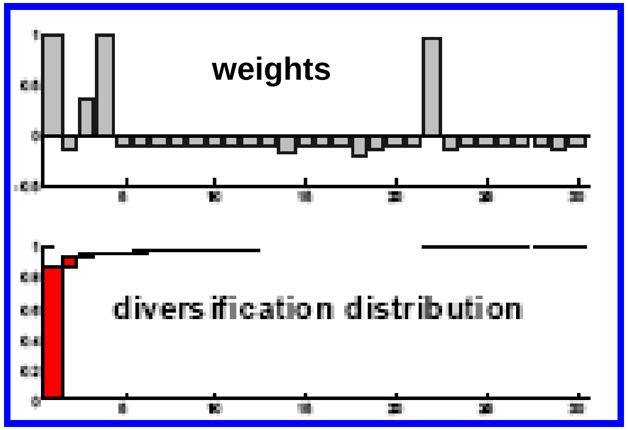
full concentration $\boxed{\mathcal{N}_{Ent} \approx 1}$



$$p_n \equiv \frac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Var}\{R_{\mathbf{w}}\}}$$ diversification distribution: “probability mass”

Mean-Diversification Frontier

Effective number of bets

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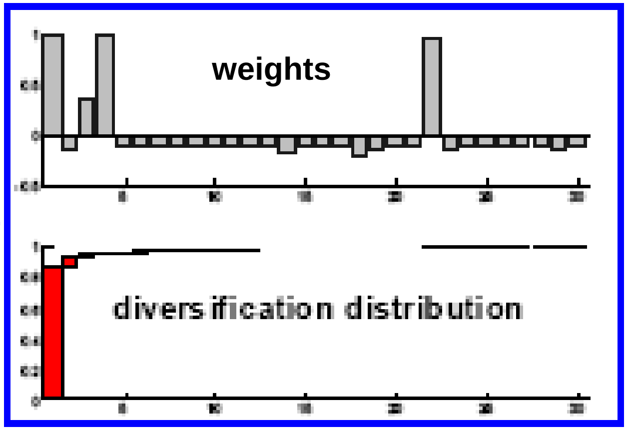
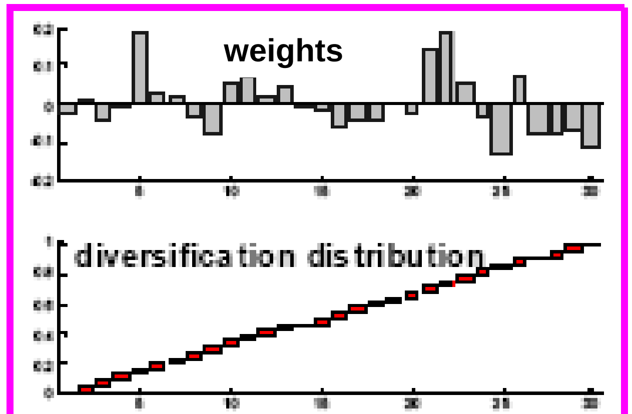
full concentration $\boxed{\mathcal{N}_{Ent} \approx 1}$

full diversification $\boxed{\mathcal{N}_{Ent} \approx N}$





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Mean-Diversification Frontier

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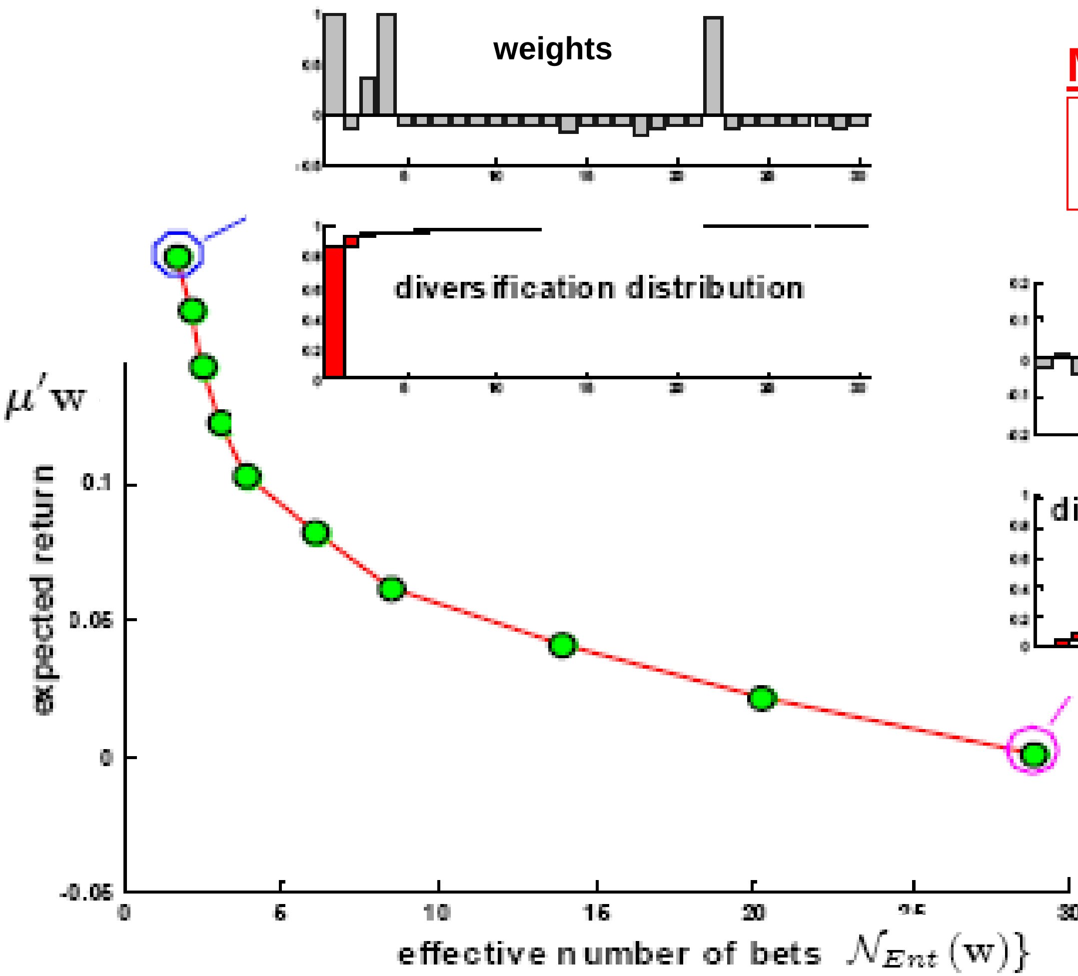
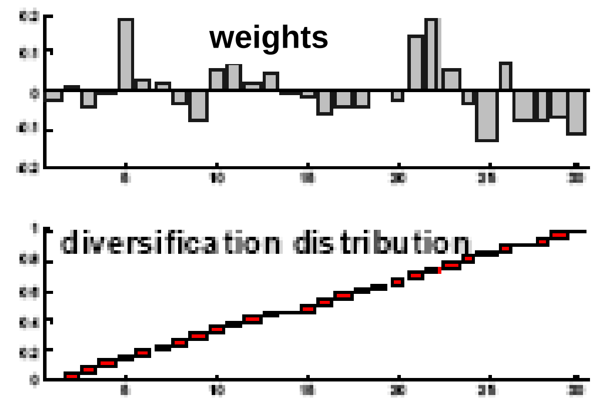
full concentration $\boxed{\mathcal{N}_{Ent} \approx 1}$

full diversification $\boxed{\mathcal{N}_{Ent} \approx N}$

Mean-diversification frontier

$$\boxed{\mathbf{w}_{\varphi} \equiv \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}} \left\{\varphi\boldsymbol{\mu}'\mathbf{w} + (1-\varphi)\,\mathcal{N}_{Ent}(\mathbf{w})\right\}}$$





Mean-Diversification Frontier

Effective number of bets

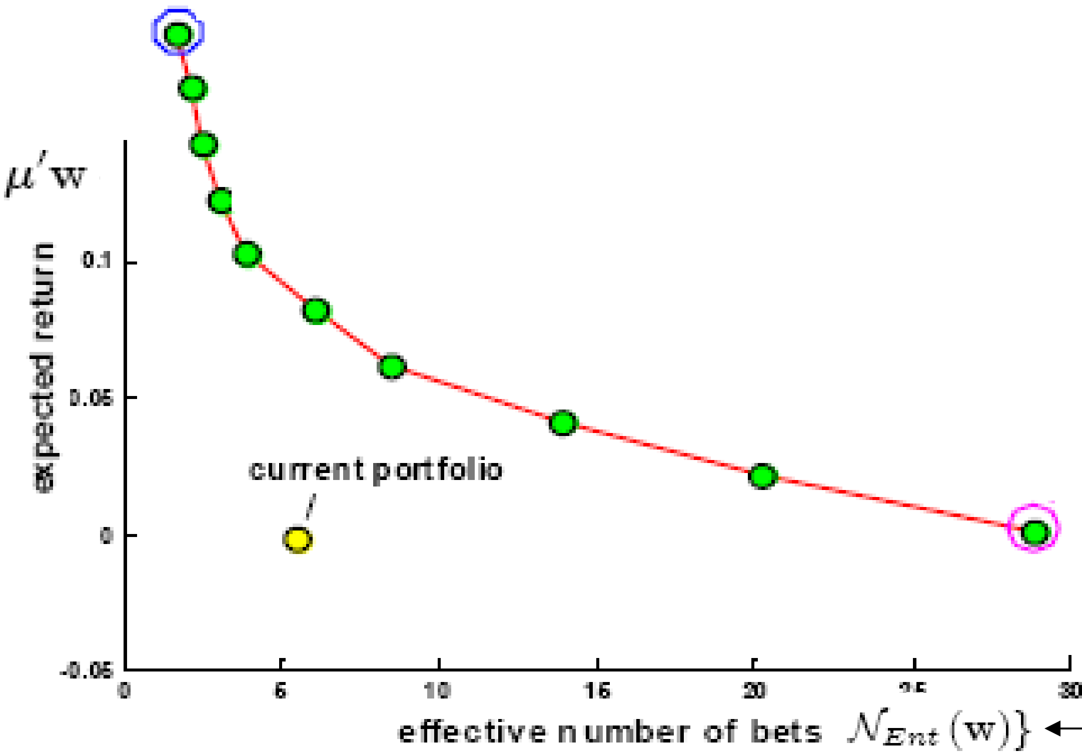
$$\mathcal{N}_{Ent} \equiv \exp\left(-\sum_{n=1}^{N} p_n \ln p_n\right)$$

full concentration $\boxed{\mathcal{N}_{Ent} \approx 1}$

full diversification $\boxed{\mathcal{N}_{Ent} \approx N}$

Mean-diversification frontier

$$\mathbf{w}_{\varphi} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \varphi \mu' \mathbf{w} + (1-\varphi)\, \mathcal{N}_{Ent}(\mathbf{w}) \right\}$$



Allocation in terms of
original portfolio weights

not principal portfolios

 Mean-Diversification Frontier

Effective number of bets

$$\mathcal{N}_{Ent} \equiv \exp\left(-\sum_{n=1}^{N} p_n \ln p_n\right)$$

full concentration $\boxed{\mathcal{N}_{Ent} \approx 1}$

full diversification $\boxed{\mathcal{N}_{Ent} \approx N}$.

Transaction costs

$$\boldsymbol{\mu}'\mathbf{w} \mapsto \boldsymbol{\mu}'\mathbf{w} - \mathcal{T}\left(\mathbf{w}, \mathbf{w}_{cur}\right)$$

Non linear, non-continuous function of current and target portfolio

Mean-diversification frontier

$$\mathbf{w}_{\varphi} \equiv \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}} \left\{\varphi\boldsymbol{\mu}'\mathbf{w} + (1-\varphi)\,\mathcal{N}_{Ent}\left(\mathbf{w}\right)\right\}$$

Mean-Diversification Frontier

Effective number of bets

$$\mathcal{N}_{Ent} \equiv \exp\left(-\sum_{n=1}^{N} p_n \ln p_n\right)$$

full concentration $\boxed{\mathcal{N}_{Ent} \approx 1}$

full diversification $\boxed{\mathcal{N}_{Ent} \approx N}$

Transaction costs adjusted mean-diversification frontier

$$\mathbf{w}_{\varphi} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{\varphi\left(\mu'\mathbf{w} - \mathcal{T}\left(\mathbf{w}, \mathbf{w}_{cur}\right)\right) + (1-\varphi)\,\mathcal{N}_{Ent}\left(\mathbf{w}\right)\right\}$$

Mean-Diversification Frontier

Effective number of bets

$$\mathcal{N}_{Ent} \equiv \exp\left(-\sum_{n=1}^{N} p_n \ln p_n\right)$$

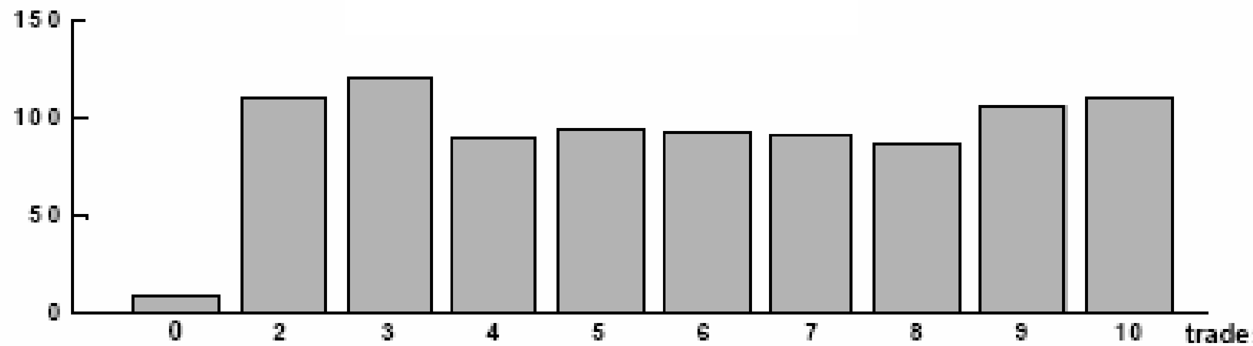
full concentration $\boxed{\mathcal{N}_{Ent} \approx 1}$

full diversification $\boxed{\mathcal{N}_{Ent} \approx N}$

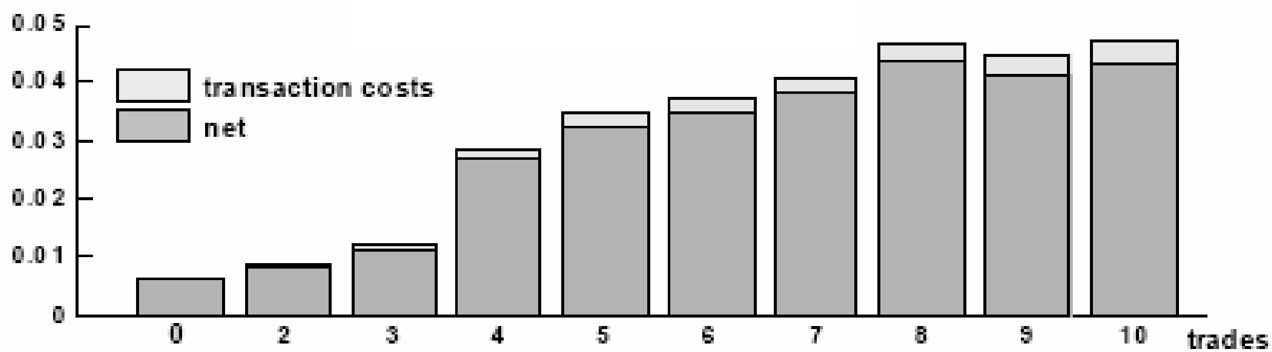
Transaction costs adjusted mean-diversification frontier

$$\mathbf{w}_{\varphi} \equiv \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}} \left\{\varphi\left(\boldsymbol{\mu}'\mathbf{w} - \mathcal{T}\left(\mathbf{w}, \mathbf{w}_{cur}\right)\right) + (1-\varphi)\,\mathcal{N}_{Ent}\left(\mathbf{w}\right)\right\}$$

Effective number of bets

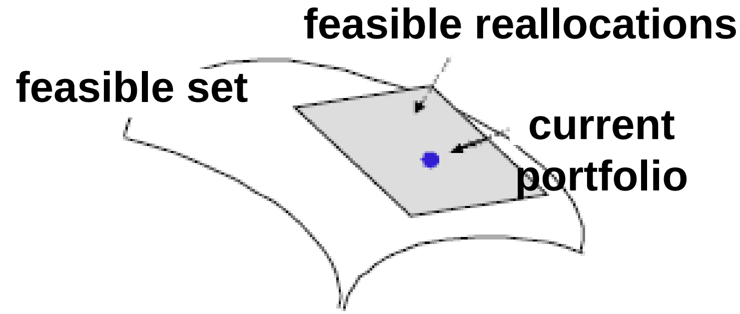


Expected return

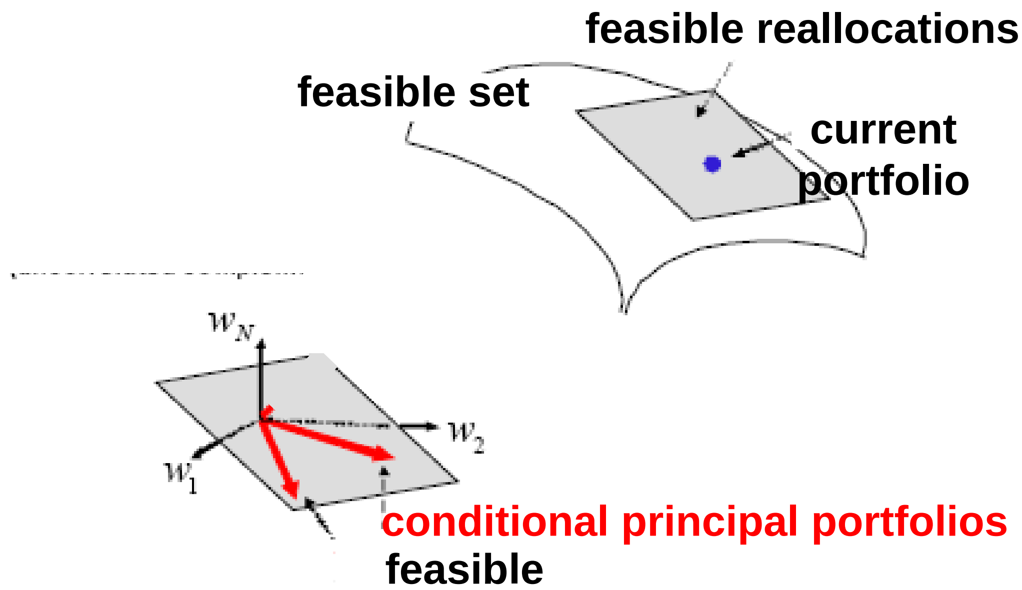


Constraints

$$\underset{K \times N}{\mathbf{A}} \underset{N \times 1}{\Delta \mathbf{w}} \equiv \mathbf{0}$$



Conditional Analysis



Constraints

$$\underset{K\times N}{\mathbf{A}}\,\underset{N\times 1}{\Delta\mathbf{w}} \equiv \mathbf{0}$$

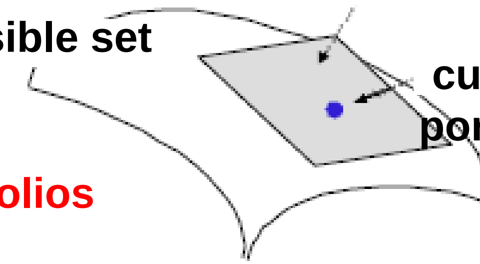
Conditional PCA

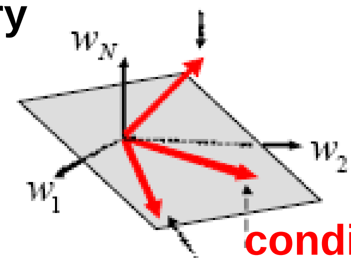
Feasible trades

$$n = K+1,\ldots,N$$

$$\mathbf{e}_n \equiv \underset{\mathbf{e}'\mathbf{e}\equiv 1}{\operatorname{argmax}} \left\{\mathbf{e}'\boldsymbol{\Sigma}\mathbf{e}\right\}$$

such that $\begin{cases} \mathbf{e}'\boldsymbol{\Sigma}\mathbf{e}_j \equiv 0 \\ \text{for all existing } \mathbf{e}_j \\ \boxed{\mathbf{A}\mathbf{e} \equiv \mathbf{0}} \end{cases}$





Constraints

$$\underset{K\times N}{\mathbf{A}}\Delta\underset{N\times 1}{\mathbf{w}} \equiv \mathbf{0}$$

Conditional PCA

Feasible trades

$$n = K+1,\ldots,N$$

$$\mathbf{e}_n \equiv \underset{\mathbf{e}'\mathbf{e}\equiv 1}{\operatorname{argmax}}\{\mathbf{e}'\Sigma\mathbf{e}\}$$

such that $\begin{cases} \mathbf{e}'\Sigma\mathbf{e}_j \equiv 0 \\ \text{for all existing } \mathbf{e}_j \\ \boxed{\mathbf{A}\mathbf{e} \equiv \mathbf{0}} \end{cases}$

Complementary, unfeasible trades

$$n = 1,\ldots,K$$

$$\mathbf{e}_n \equiv \underset{\mathbf{e}'\mathbf{e}\equiv 1}{\operatorname{argmax}}\{\mathbf{e}'\Sigma\mathbf{e}\}$$

such that $\begin{cases} \mathbf{e}'\Sigma\mathbf{e}_j \equiv 0 \\ \text{for all existing } \mathbf{e}_j \end{cases}$

- **Article:**
  Attilio Meucci, "Managing Diversification"
  ***Risk* - May 2009**
  extended version available at http://ssrn.com/abstract=1358533

- **MATLAB examples:**
  MATLAB Central Files Exchange (see above article)

- **This presentation:**
  www.symmys.com > Teaching > Talks