“You don’t know the power of the dark side!”
Darth Vader,
The Empire Strikes Back
A market center is a venue in which you can execute orders to buy or sell shares.

Dark Pools are a subset of all market centers.

In the past, every market center that was not lit was considered dark.

In order to understand what lit markets are, we need to survey how the US equity market developed over the years.

In 1975, to encourage transparency, efficiency, and innovation the congress asked the SEC to establish a National Market System.

In response, the SEC established the National Market System (NMS).

The NMS is responsible for distributing the best bid/ask quotes from all the exchanges and all the trades that occur in every market center.
To encourage innovation, the act states that if you are not too big, you can implement almost any idea you have.

In 1998, regATS was implemented. Essentially, making a clear definition of what a big market center is; making it easy to create non-regulated market center; allowing ATSs to limit participants, etc.

In 2007, regNMS was established. Making the inside quote protected. Starts the latency arm race.

In essence, lit markets are markets the distribute their firm quotes to the consolidated tape.

Dark Pools do not distribute the state of their order book to the consolidated tape.
Currently, there are 30+ Dark Pools that trade about 10-20% of the overall volume in the US Equity market.

Today, the distinction between dark and lit markets is blurred.

Lit markets are going dark, Dark Pools are going lit.

Some Dark Pools are distributing Indication Of Interest (IOI) messages to their clients.

An IOI message reveals some information about the orders in the pool.

For example, a message might contain symbol, symbol and side, or symbol, side and size.

IOI is very similar to a firm quote, but it is distributed only to a subset of the market participants, promoting two tier market.

Lit destinations support hidden orders which are just like dark pool orders.
Originally, Dark Pools were created to answer a need.

Probably the first dark pool was the floor brokers at NYSE during the mid seventies.

Suppose you want to buy one ADV of a stock. What are your options?

- Trade VWAP over the day. Large impact, predictability, etc..
- Trade over 10 days, 10% of the order each day. A lot of risk.
- Trade slowly, wait for someone that needs to sell one day ADV. Trade with him the remaining quantity.

In the past, a NYSE floor broker would get the order and would work it slowly.

The full order was not published as a firm quote.

The broker would trade the full quantity once a natural contra have been found on the floor.
Over the last decade or so, the US population have moved toward a central money management structure.

401K, Pension funds, Mutual funds have increased in size. As a result, each money manager has to trade at an ever increasing size.

Instead of many uncorrelated small orders, we get correlated large orders.

Maybe getting out of topic, but it also seem that relaying on sell side information have resulted in an increased correlation in the buy side.

This development has resulted in the popularity of Algorithmic Trading which provides a solution for trading large sizes.

With sizes becoming even larger, Dark Pools were born to provide environment in which people can rest big orders and wait for a contra without reveling their intentions to the overall market.
Dark Pools support many different order types and attributes.

- **Mid-Point**: The order is pegged to the prevailing mid-price of the NBBO quote.

- **Peg order**: The order is pegged to any point inside the spread, e.g., ask, bid.

- **Minimum Quantity**: Specifying not to interact with orders smaller than this quantity.

- **Persistent minimum quantity**: The minimum quantity is forced throughout the life of the order and not just for the first fill.

- **Limit Price**: Do not execute the order at a price inferior than the limit price.

- **Do not interact with**: Excluding various counter parties.

- **Do not send IOI**: 
- **Block oriented Dark Pools**: pools that enforce a minimum size for each order.
  - Liquidnet, Pulse, AQUA to name a few.
  - The minimum order size ranges from 5,000 to 50,000+ shares.
  - Block oriented dark pools were created to ensure that trades occur in big sizes, avoiding the need to trade large quantities in the open market.
  - This pools also use blotter scraping technology.

- **Streaming Liquidity Pools**: These pools do not enforce any minimum order quantity.
  - NYFIX, SigmaX, Level, etc.
  - Allow to trade small quantities.
- **Schedule Cross**: A schedule cross destination performs an auction at pre-specified times.
  - POSIT, IDX.
  - An auction is performed every, say hour. Market participants send their orders prior to a cross. A match occurs and fills are sent out.
- **Continues Cross**: The pool operates using a double auction structure. A match can occur any point in time. CBX, LCX, MLXN.
- **Hybrid Markets**: Pools that offer exotic crosses.
  - BLX.
  - An auction is performed once enough shares can be crossed.
- **Forward Looking cross**: Crosses that are priced based on forward looking benchmarks, say VWAP, Close, ...
- **Automated Networks**: All orders are firm orders. If two parties has opposing orders they are matched and reported to the tape.

- **Negotiated Networks**: Orders are not firm. If two parties has opposing orders they get the option to confirm the trade, or to negotiate a price with each other. Blotter scraping.

- Negotiated networks allows a trader to back-out of a trade and gain information about the existence of other orders.

- **Hybrid Networks**: A combination of firm and negotiated orders are used in the pool.
Networking effect is essential for transparent and efficient price formation process.

NMS and regNMS resulted in one integrated US market, albeit very fragmented. (Worst of all worlds.)

How come we have 30+ Dark Pools and there is no consolidation in sight.

In the last ten years the taker/provider model became the dominant pricing model in the US Equity market.

As a result, liquidity takers’ cost of trading has increased significantly.

Dark Pools allow a broker/dealer to internalize flow, i.e., two clients of the same broker can easily trade directly with each other, without the need to pay taker fees and at a superior prices (mid instead of the inside.)
Many broker/dealers operate a Dark Pool. Rumors are that the big banks are internalizing from 30% to 90% of their flow.

Different Dark Pools charge different fees.

Many of these Dark Pools are connected in a complex web of connections.

Different pools charge very differently.

Pools that provides economic benefit, like large block cross charge 1c per share or more.

Pools that aims to avoid the taker fees will charge much lower fees.
The first problem one is facing is how to trade in the presence of Dark Pools and exchanges.

Dark pools provide us the option to trade opportunistically very large quantity at no cost, i.e., impact.

How can we combine optimal trading in exchanges and take advantage of Dark Pools.

Let us see the solution for the single asset problem.

Assume we would like to liquidate $X_0$ shares.

Assume that trading time is discrete, and we have $N$ time slots at which we can trade. $x_i$ is the number of shares we trade at time $i$.

Denote by $f(x_i)$ the cost of trading $x_i$ shares in the open market.

With probability $p$, there will be infinite amount of liquidity in the dark pool, and the cost of trading in the dark pool is free.
- Our aim is to find the optimal policy, $x_{opt} = [x_0, ..., x_{N-1}]$ such that

$$x_{opt} = \arg \min_{x_0, ..., x_{N-1}} E \left\{ \sum_{i=0}^{N-1} \lambda X_i^2 \sigma + f(x_i) \right\}$$

$s.t.$

$$\sum_i x_i = X_0$$

$$X_i = \begin{cases} X_{i-1} - x_i & 1 - p \\ 0 & p \end{cases}$$

- The assumption of infinite liquidity and zero cost will result in a solution in which whatever we do not trade in the open market, we should place at a dark pool.
Solving this type of problem can be accomplished via dynamic programming.

Denote by $J_i(x_0, \ldots, x_{i-1}, X_i)$ the cost of optimal trading policy after trading $x_0, \ldots, x_{i-1}$.

It is easy to see that the remaining cost is independent of our “path” to $X_i$ and depends only on the remaining shares to trade.

$$J_i(x_0, \ldots, x_{i-1}, X_i) = J_i(X_i).$$

We can now solve this problem “backward”

It is easy to see that, $x_{N-1} = -X_{N-1}$. (We need to finish the trade.)
Given that we are at $N - 2$ and we have $X_{N-2}$ shares to trade. How much we should trade?

$$J_{N-2}(X_N - 2) = \arg\min_{x_{N-2}} \lambda X_{N-2}^2 \sigma^2 + f(x_{N-2})$$

$$+(1 - p)J_{N-1}(X_{N-2} - x_{N-2})$$

- We can now easily solve for $x_{N-2}$.
- We can continue this process backward until we find $x_0$. 
What would you expect the solution to be?

- Assume that $p = 0$.
- Optimal execution as we know it.

- Assume that $p = 1$

- $x_0 = 0$ everything should be placed at the dark pool. For sure it is going to be executed.

- Assume $Np \gg 1$

- Very back-weighted trajectory. With high probability, we will be able to execute at the dark pool the whole quantity during the execution period.

- Assume $Np \ll 1$.

- Small deviation from optimal execution.
Figure: Optimal Liquidation of a security in the presence of dark pool exposure. Risk aversion. (Optimal Liquidation in Dark Pools - Kratz)
- The reality of dark pools is more complicated.
- In reality, liquidity is not infinite.
- This is easy to handle, as our averages can be taken over all the possible liquidity in the pool.
- In reality, trading in dark pool is not free. There is an impact for trading in dark pool.
- We can add to our equations the impact of trading in dark pool.
- No one came with a clear model for the impact of trading in Dark Pools yet.
How come there is impact for trading in Dark Pools?
You should argue that the two participants cancel out the information transmitted to the market when they trade with each other.

The reality of Dark Pools is more complicated.
The premise of Dark Pools was to allow traders to trade large orders without leaking information to the market.
I claim that this is true if and only if both sides of the trade have the same order on opposite sides.

Assume that you have a 50,000 shares buy order of GLG in a dark pool.
Out of a sudden, you are fully executed.
Probably there is more than 50,000 shares in the contra order.
These excess shares have to be traded somewhere, and they will drive the price down.
Buyer remorse.
Assume that you have a 50,000 shares buy order for GLG in a dark pool.

Someone send a 100 sell order for GLG and the order is executed against your order.

The contra learned that their is a “big” order to buy.

He can now wait until you are forced to buy in the open market and move the price in his favor.

The existence of size discrepancy entail information leakage with every execution.

This, in turn, leads to adverse selection, i.e., you regret that your were executed.

People continuously look for ways to identify the existence of a large orders resting in Dark Pools and “game” the order, i.e., earn close to riskless profit.
Gamer is a common name for a market participant that infer the existence of an order at a Dark Pool.

Upon learning about the order a gamer engage in a trading strategy that results in a riskless or close to riskless profits.

How can this happen given the EMH?

There are various ways to accomplish that: quote manipulation, pinging, front running, price moving, and more.

Let’s examine each case separately.

Throught this section we will assume that you have 50,000 shares buy order for GLG.
- Assume that I have a sell order for 50,000 shares of GLG.
- The market is 36.34x36.46.
- If I send the order to the pool, we will trade at \( \frac{36.34 + 34.46}{2} = 36.4 \).
- Can I do better?
  1. Send a buy order for 100 shares to say, NASDAQ, at 34.45.
  2. Wait for a millisecond for the quote to update to 36.45 \times 36.46.
  3. Send the sell order to pool. Be executed at 36.455.
  4. Cancel your limit order.
- We sold the stock 4.55c higher than we “should” have.
- This procedure is one type of quote manipulation.
- It is also prevalent with plain vanilla algorithms and it is probably the oldest form of gaming.
Figure: Example of quote manipulation. (What is a quote - O’Hara)
Assume I have no intention to trade GLG but I learned about your order.

Equipped with this knowledge I can make almost riskless profit.

1. Quickly buy 3,000 shares in the market and move the price by 3c.
2. Post a quote to lock the market at 1c spread.
3. Wait for the NBBO to update to 34.48 × 34.49
4. Sell 3,000 shares in the dark pool.
5. Remove my limit.
6. Wait for the price to revert.
7. Return to 1.

In each round we can make around 1-2c per share almost riskless profit.

People will refer to this method as classical gaming.
Types of Gaming

- Information Leaked
- Gaming
- Pushing the price
Figure: Example of Gaming via pinging and front running
Assume I have no intention to trade GLG but I learned about your order.

I can still make almost riskless profit from this knowledge.

1. Buy at a constant high rate shares of GLG.
2. Create large continues temporary impact.
3. Every time I accumulate, say 3,000 shares, I will sell them in the pool.

This method is called front-running.

In this case, the price moves up very rapidly and exhibit price reversion at the end.
Types of Gaming

- Price

- Information Leaked

- Impact, Adverse Selection, and Gaming in Dark Pools
Figure: Example of Gaming via pinging and front running
Engaging in gaming requires to detect dark orders. How this can be accomplished?

Some market participants partner with a Dark Pool to provide liquidity in the pool.

This is done via sending IOIs to the partner, which in turn can learn about the existence of the order.

The awareness of market participants to this type of conflict of interest reduced this case substantially.

Pinging Dark Pools. I can send a small IOC order, say 100 shares, to the pool. If the order is executed, I can learn about the existence of a big contra order in the pool.

In pinging Dark Pools, the gamer takes some risk, but this risk is much smaller than the value of the information gained.
- Even better way is to use riskless IOC.
- Dark Pools allow to specify a flag that the order will not execute against resting orders.
- Send such an order to a dark pool. If the order is canceled back, you know that there is an order on the other side (why?)
- If the order is not canceled back you will cancel it yourself.
- Tape reading. From observing the trades on the tape you can infer the existence of orders.
- Adverse selection and gaming is not the only mechanisms through which impact is created.
- Consider an order to buy 50,000 shares of GLG which is distributed among many Dark Pools and hidden orders in lit market.
- In essence, any market order to sell is executed against your buy order.
- Market Maker’s limit sell orders are executed against incoming buy market orders.
- As time progress the market maker accumulates short position, and starts to update its quote to solicit sellers.
- Your trade creates impact in a non-direct way by causing other market participants to adjust their quotes.
Anti-gaming in dark pools can be divided into the pool own measures and the buy-side measures.

Dark pools use various methods for avoiding gaming and information leakage.

1. **Large Blocks pools.** By enforcing large minimum quantity for each order the pool prevents the gamer ability to detect large orders without taking significant risk.

2. **Limiting participants.** As was mentioned, Dark Pools can prevent access. Some Dark Pools prevent certain type of market participants from their pool.

3. **Participants analysis.** Some pools that send IOIs will require the other side to act on an IOI with high probability (85%). This limit the IOI recipients from cherry picking trades.

4. **Statistical analysis.** One can conduct rigorous analysis and detect participants that consistently “make” money.
Buy side traders employ their own anti-gaming algorithms,

- Minimum quantity. By specifying that they want to interact with “large” orders the trade can avoid most of the predatory trading as described.
- Using minimum quantity can result in a significant reduction in the volume one is exposed to.
- Using minimum quantity “bend” the risk/reward of the gamer, requiring the gamer to commit more capital and take more risks.
- Limiting participants. Some pools allow you to specify the possible contra-parties for your trades. For example, you can opt-out from interacting with high frequency traders.
Relative movement. We can use a sector ETF to price the stock and put limit prices based on these values. For example,

\[ p_l(t) = \beta p(0) \log \frac{p_{ETF}(t)}{p_{ETF}(0)} + \delta \sqrt{t} \sigma_r \]

where \( \beta \) is the \( \beta \) of the stock to the sector ETF, \( \sigma_r \) is the residual volatility after subtracting the effect of the sector from the stock returns.

Relative value method works well in large cap stocks for which the CAPM style model has higher \( R^2 \).

For small cap stocks, ones for which \( R^2 \) is low, this method essentially will not result in a tight limit prices.

Using the stock price process to estimate an efficient price and not allowing to deviate from this price. This can be as simple as taking a trailing VWAP/TWAP prices to complex model that take into consideration inventory, price pressures, etc.
As was discussed, there are 30+ dark pools.

Volume is not divided equally, and in essence, it is concentrated on a name by name basis.

If BarCap has a big order in name X there is going to be a lot of volume and activity at LCX.

Dark Pool aggregators provide access to all the pools simultaneously and create a one point access to the fragmented liquidity.

A dark pool aggregator receives an order and split it between the various pools.

As the algorithm get fills it adjust the allocation to reflect the level of activity in the various pools.
The first generation of dark pool aggregators were very simple. In essence, they used what we refer to as a bathtub allocator.

The algorithm tried to rebalance between the pools keeping an equal number of shares in each pool.

Later more sophisticated ad-hoc procedures were devised for this algorithms.

What a dark pool aggregator objective function is?

Many would argue, that an aggregator needs to maximize \( E_q\{\min(q_i, x_i)\} \), the execution rate per unit time.

This problem can be solved via a greedy algorithm or water filling algorithm very effectively and it depends on \( F_{X_i}(x_i) \).

Current aggregators solve a much more complex problem.
Suppose you would like to trade in dark pools but you do not want to leak information.

As was mentioned earlier, you might use minimum quantity in order to “avoid” some incoming orders.

What minimum quantity should you use?

The minimum quantity should depend on the current market condition, trading objectives and more.

We can define a trading goal in shares per unit time and look for the highest minimum quantity that still achieve this trading goal.

In particular, we are looking to solve

$$\max_{q_{\text{min}}} E_{q_{\text{min}}, q}\{\min(q_i, x_i)I_{x_i \geq q_{\text{min}}}\} > T.$$ 

This problem is by far harder to solve but algorithms to solve it an an efficient way exists.
Trading efficiently in Dark Pools requires navigating a very complex eco-system, one that combines liquidity, predatory trading, impact, and more.

At this point there is no one model that address all these aspects.

As such, in order to provide superior performance, one needs to address each problem independently.

One aspects is to identify toxic pools. i.e., pools that provide inferior performance.

The ability to detect these toxic pools is essential for providing superior performance.

There are various methods to do so, and in what follows I am going to describe a few.
Quote manipulation strategies results in adverse selection.

You are always executed at the pick of a local price move.

How can we detect a pool whose executions are unfavorable in this respect.

Suppose you calculate the TWAM (Time Weighted Average Quote) around a fill.

\[
\bar{p} = \int_{t-30}^{t+30} p_m(\alpha) d\alpha
\]

If the average price around a fill is equal to the fill price, one can consider the fill price as a fair price.

If the average price for a fill is systematically higher than the TWAM price, one can conclude that the fill price was unfavorable.
Figure: Short Term Adverse Selection. (Understanding and Avoiding Adverse Selection in Dark Pools - Saraiya Mittal)
**Figure:** The benefits of using TWAM for indicating short term adverse selection. (Understanding and Avoiding Adverse Selection in Dark Pools - Saraiya Mittal)
Using the TWAM idea one can construct a measure that compares destinations’ quality by comparing their TWAM shortfalls.

It can also be used to identify instances of gaming.

Note that the TWAM metric is not casual. It is an ex-post measure.

If the ex-ante portion of this measure was useful in identifying adverse selection, you could use it as your anti-gaming measure.

It is the reversion that differentiate the valid price moves from adverse selection.
Figure: The fill metric ability to differentiate bad trades.
How can we detect pools that are home to front runners.

We can observe the behavior around a fill in the various pools.

This can be accomplished in several ways.

One can align all the fills from a pool and look for differences between the pools.

In essence, we can average the shortfall before and after the fill cross sectionally per destination.

If fills occur at random times at random pool (not necessarily uniformly) we should not observe different behavior across pools.

If one pool is more likely to be used by front runner, we expect to see at that pool different temporal behavior of the shortfalls.

In the following figure we depict the results of such study.
Figure: Returns around fills in various destinations.
So far all our method were based on analyzing the fills ex-post.

Ex-ante methods provide very interesting results and insights.

Suppose for every task, we decide not to use one of our 30+ destinations at random.

What can we learn from such an experiment?

For every destination we have two populations. One that used and one that didn’t use the destination.

Since our decision which order belong to which destination was taken in random, there is no bias in term of the orders characteristics in each group.

One can conduct a test to decide whether the average shortfall in the two populations is the same.
- If the result of this analysis indicates that the shortfall when destination $X$ was used is significantly higher than when the destination was not used, we should stop using the destination.
- Often classical tools like $T$–test have a higher false alarm then predicted.
- Bootstrap testing methodologies are usually used for conducting the tests.
Figure: Relative performance of various destinations.
You send a hidden mid-peg order to a Dark Pools.

You expect the fill to occur at the mid point.

Sometimes programmers make mistakes or there are other exogenous reasons that result in a trade on the wrong side.

The following figure depicts the distribution of execution location for three different destinations.

In the figure, we depict the distribution of the distance from the mid (in terms of bid-ask spread).

In the figure, one can exhibit that errors are not random but are more often not in your favor.
Figure: Bid-Ask shortfall for three different destinations
Dark Pools and dark orders offer unique opportunity for liquidity.

Using and interacting with dark pools is a huge challenge.

Problems like trade-out, allocation, anti-gaming, adverse selection, pool vetting are researched extensively in the industry.

Those are all open problems that require more rigorous approaches and solutions.

The lack of data makes it hard for researchers to conduct innovative research.
Thank You