Monotonicity of tail risk under temporal aggregation in GARCH$(1,1)$ models

The square-root-of-time rule is often used in financial practice for temporal aggregation of risk. However this requires that returns are conditionally serially uncorrelated, which is not satisfied in practice due to volatility clustering. A more realistic model is GARCH(1,1) which reproduces the volatility clustering property of financial time series.

It was shown by Drost and Nijman that the class of weak GARCH models is closed under temporal aggregation, implying that the n-step aggregated GARCH(1,1) model is also GARCH(1,1) with appropriately modified parameters $(\alpha_n, \beta_n, \omega_n)$. The resulting conditional distributions can deviate appreciably from the square-root-of-time rule.

We study the tail asymptotics of the unconditional distribution of returns under temporal aggregation. The unconditional distribution of the n-step returns r_n has power-law tails $\mathbb{P}(r_n > x) \simeq x^{-\kappa_n}$ with a calculable exponent κ_n. We derive a criterion for strict (increasing) monotonicity of the map $n \to \kappa_n$, and show that it is satisfied in a large region of the GARCH(1,1) parameter space, for a wide choice of innovations. We also prove the existence of exceptions to the monotonicity criterion in a small corner of the parameter space. (work with P. Glasserman and Q. Wu)