

# Where is the Option? Prepayment Modeling of MBS



April 30, 2007

# The Size of the U.S. Mortgage Market

## Home mortgages

\$10 trillion outstanding

\$2.5 trillion originated in 2006

## Mortgage-backed Securities (MBS)

\$6 trillion total outstanding

Of which \$4 trillion are Fannie Mae, Freddie Mac,  
and Ginnie Mae ("Agency" MBS)

\$4.5 trillion Treasury securities outstanding

# Conventional U.S. Mortgages

## Offered in standardized structures

Maturity: 30-year or 15-year

Rate: Fixed, ARM, or hybrid ARM

Principal payment: amortizing or interest-only

Borrower can reduce rate by paying points, or  
avoid closing costs by accepting a higher rate

## Prepayable at any time, without penalty

Refinancing entails transaction cost

## What Is an Agency MBS?

Lender/owner of mortgages wants to securitize pool

Say 6% FRMs into a 5 ½% MBS

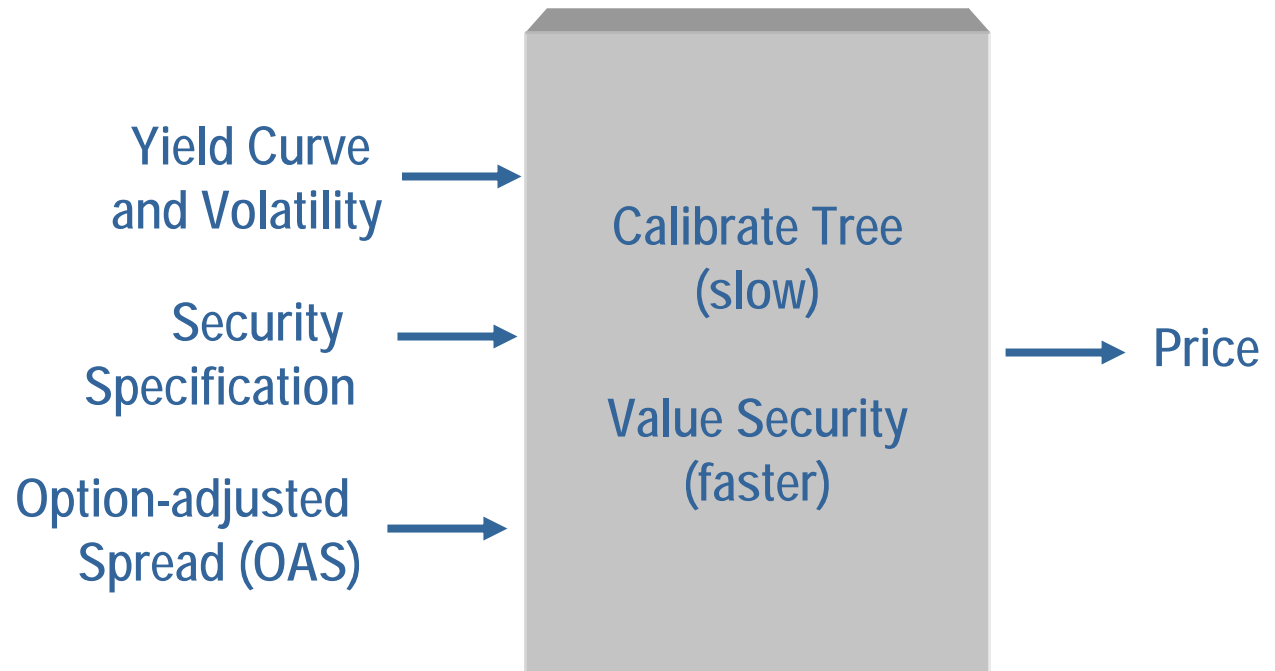
Agency overlays its guarantee and creates MBS

Principal and specified interest passes through to MBS investors

Residual interest covers servicing cost and guarantee fee



# OAS-Based Valuation of Bonds and MBS Since 1986



# Bond Valuation Speed Has Improved Dramatically

|                                   | 1990               | 2007                 |
|-----------------------------------|--------------------|----------------------|
| Processor                         | 386                | 3.0 GHz<br>Pentium   |
| Calibrate tree                    | Up to 4 hours      | 500<br>per minute    |
| Compute fair value<br>recursively | Several<br>seconds | 80,000<br>per minute |

# MBS Analysis Lags By a Mile

## Slow

At most a few 100 of valuations per minute

## Imprecise

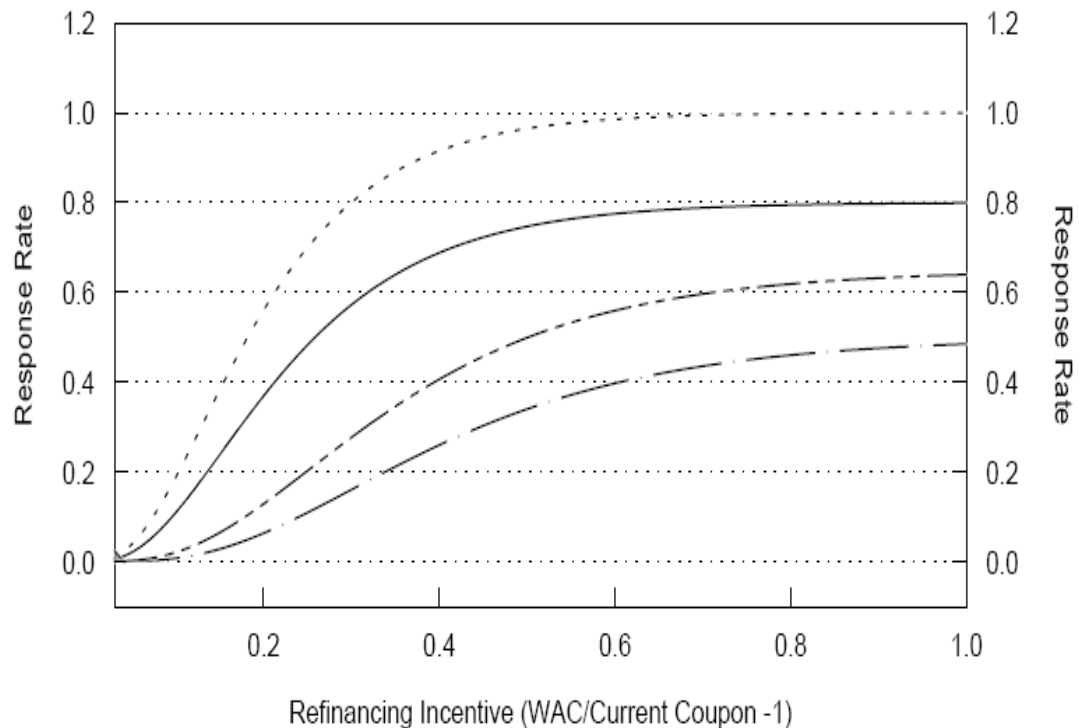
Monte Carlo simulation, rather than recursion

Prepayment models are derived by statistical analysis, rather than financial principles

Ongoing release of "new and improved" models is a predictable consequence

# Typical S-shaped Prepayment Model: Rate Depends on Refinancing Incentive

Figure 14. Refinancing Curves for Different Borrower Types



WAC Weighted-Average Coupon  
Source: Salomon Brothers Inc.

# MBS Models Can't Cope with Pertinent "What if" Questions

What if refis were optimal?

(MBS yields would increase by 10 to 15 bps)

What if borrowers gave up the right to refi?

(MBS yields would decline by 45 to 50 bps)

How sensitive are MBS prices to closing costs?

*Can prepayments be sensibly modeled  
without a command of refis?*

Flexible  
Fast ACCURATE  
Cutting-  
FLEXIBLE edge  
Accurate  
CUTTING-EDGE  
Flexible  
Fast ACCURATE  
Cutting-  
FLEXIBLE edge  
Accurate ACCU  
CUTTING-EDGE  
Flexible  
Fast ACCURATE  
Cutting-  
FLEXIBLE edge

*The right approach to MBS valuation:  
Understand mortgages; the rest will follow*

# Analytical Model of Refinancing Decision

## Mortgage: a callable amortizing bond

Represent call prices as remaining principal plus anticipated refinancing cost

e.g. 1% of remaining principal

## Mortgage rates: OAS to a benchmark curve

Use a volatility consistent with swaption vols

## Optimum: act only if value received is adequate

Provides a benchmark for sub-optimal behavior


# The Right Decision Tool Is Generalized Refunding Efficiency

$$\text{Efficiency}_{gen} = \frac{PV \text{ Savings}}{\Delta \text{ Option Value}}$$

# Refinance or Wait? Decide Like a Professional!

## Should I Refinance?

### CURRENT MORTGAGE

Years left:  


Interest rate (%):

Remaining principal (\$):

**Refinance Now?**

*patent pending*

### NEW MORTGAGE

Term (in years):  

Interest rate (%):

Discount points (%):

Upfront costs (legal fees, etc.) (\$):

New principal (\$):

*Use Custom Inputs*

## And The Answer Is...

### CASH FLOWS

Current monthly payment (\$):

New monthly payment (\$):


Savings per month (\$):

Principal remaining after 26 years (\$):


*Analysis as of 09/07/2005 11:02:14 10-Yr Treasury at 4.131%*

### RECOMMENDATION

Total savings (in today's \$):



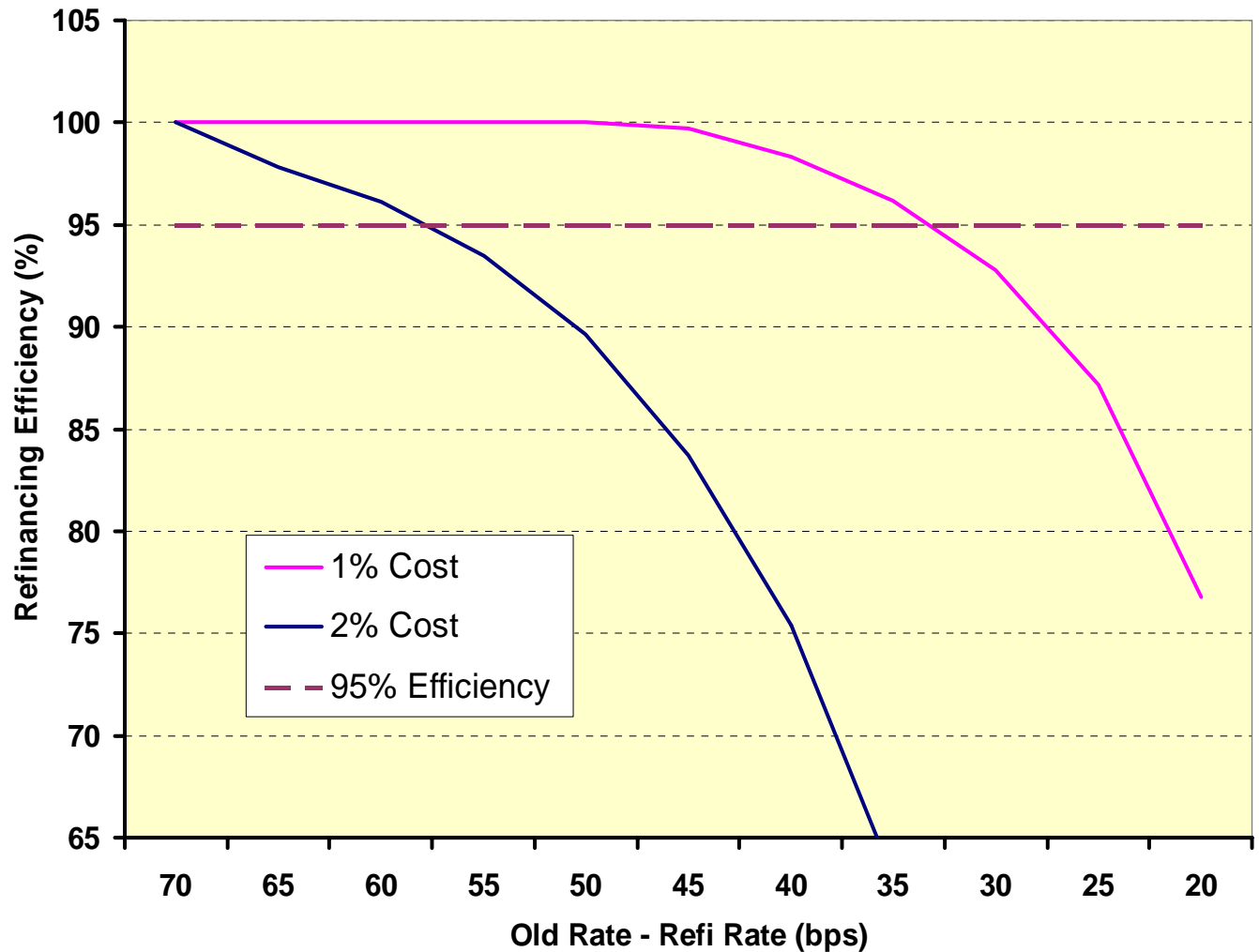
Loss of option value (in today's \$):



**Kalotay Refi Score**   
100% best. Refinancing not recommended below 90%.

**Not  
Yet!**

# Transaction Costs Impede Refinancing (And Increase MBS Value)



## Coupled *Lattice Efficiency Analysis*

Mortgage rates and MBS rates are modeled as a coupled lattice

Each lattice has its own OAS spread relative to benchmark term structure

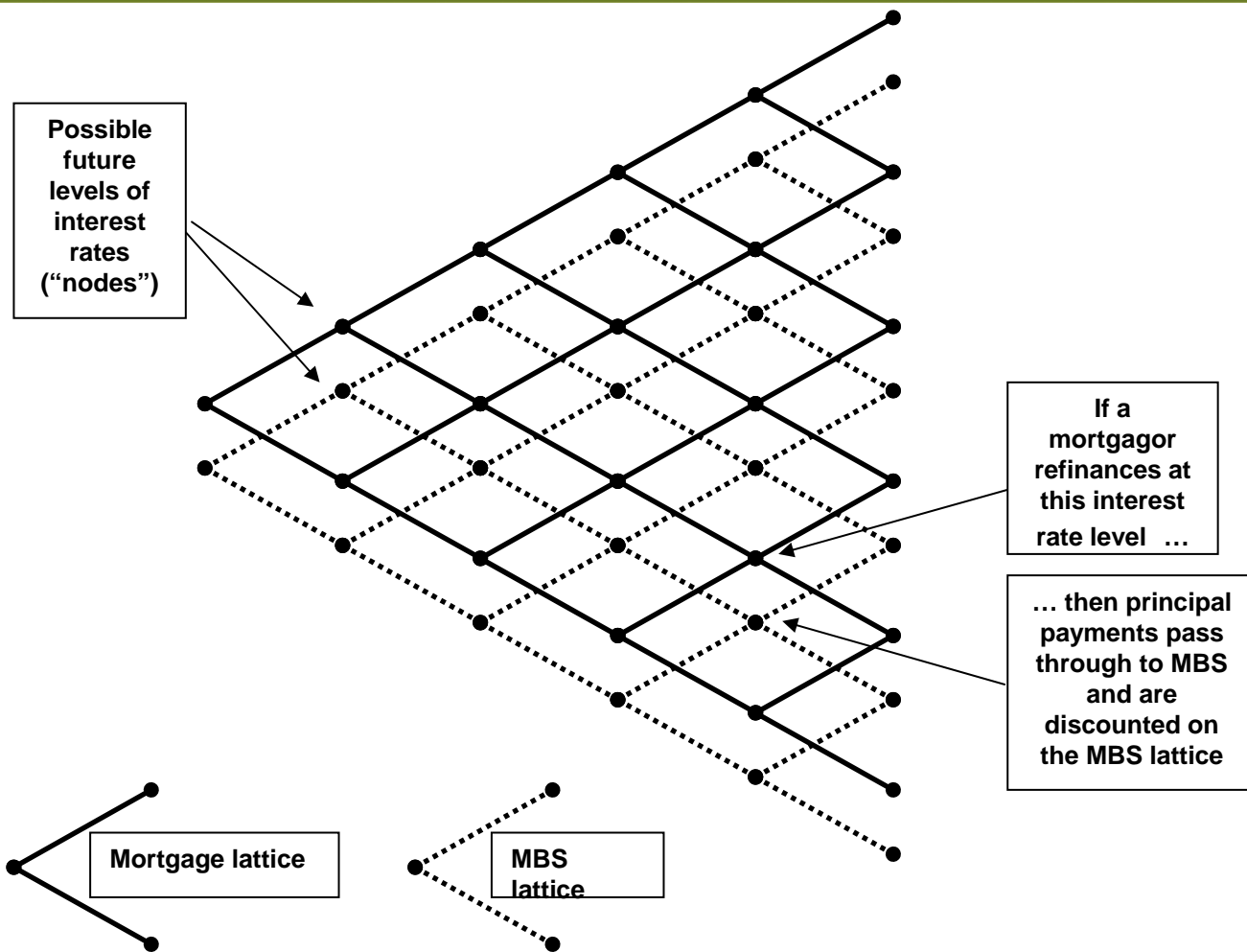
Mortgage rates determine refis

Using notion of refunding efficiency

MBS cashflows (coupon and principal) discounted on MBS lattice

*See "An Option-Theoretic Prepayment Model for Mortgages and Mortgage-Backed Securities" in References*

# Mortgage-MBS Coupled Lattice



# CLEAN™ Framework for Prepayments

A deterministic model of turnover

Say a specified annual rate

A simple but complete parametrization of interest rate driven refinancings

A wide spectrum of refinancing behavior

Pool is divided into several buckets

- Financial engineers, leapers, and laggards

For a given behavior, prediction is straightforward

Burnout is a natural consequence of the model

As “leapers” refinance, pool becomes weighted towards “laggards”

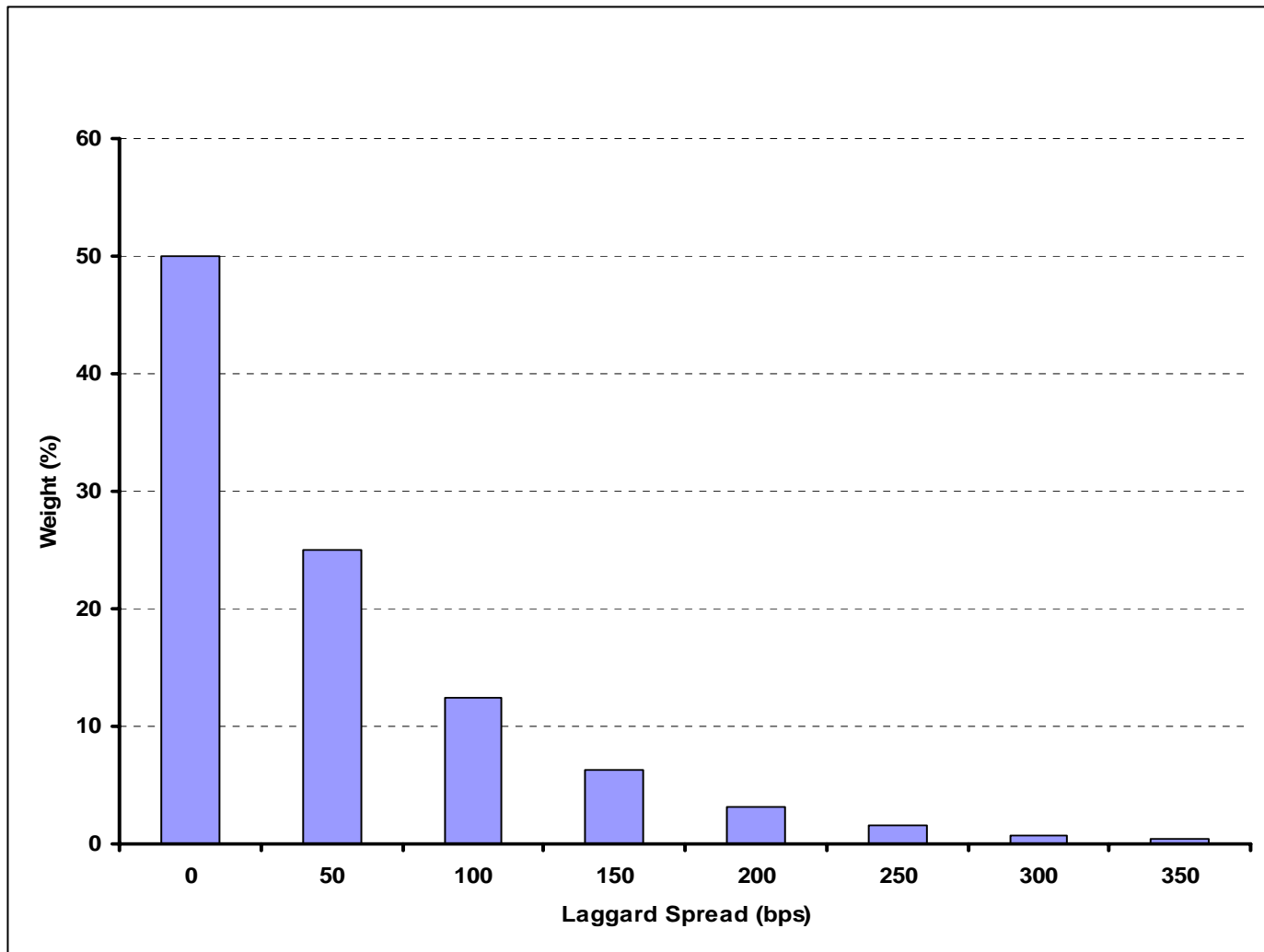
## Modeling Laggards

Each mortgagor is assigned an “imputed” coupon representing refinancing behavior

Refinancing is triggered when a Financial Engineer would refinance a mortgage with the imputed coupon

A 7% mortgagor whose imputed coupon is 6% is a 1% Laggard

# Laggard Distribution Used In Analysis Below (Same as in Paper)



# Baseline Inputs for Market Testing

September 5, 2006

## Environment:

Reference yield curve

Swap curve

Interest rate volatility

14%

## Mortgagors:

Turnover rate

8%/year

Mortgagor OAS

70 bps

Refinancing cost (% principal)

1%

Laggard distribution

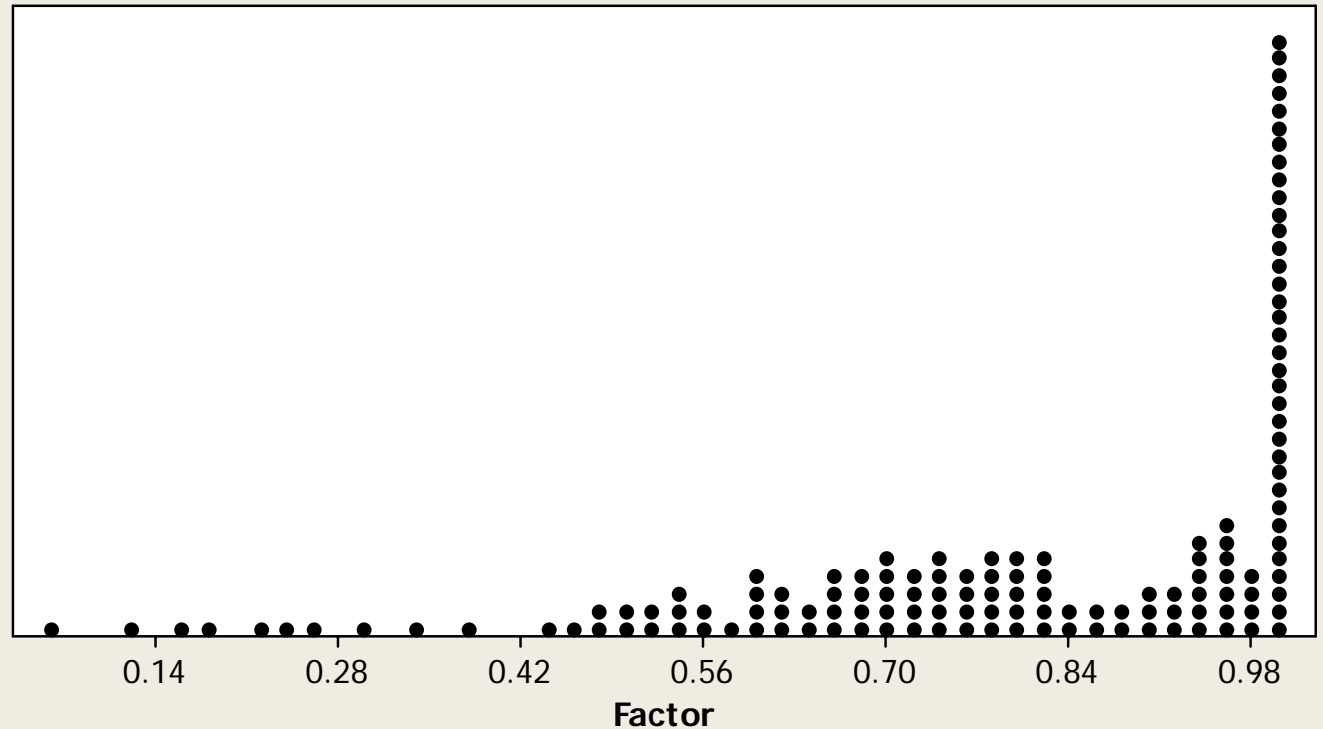
Original\*

MBS: OAS

5 bps

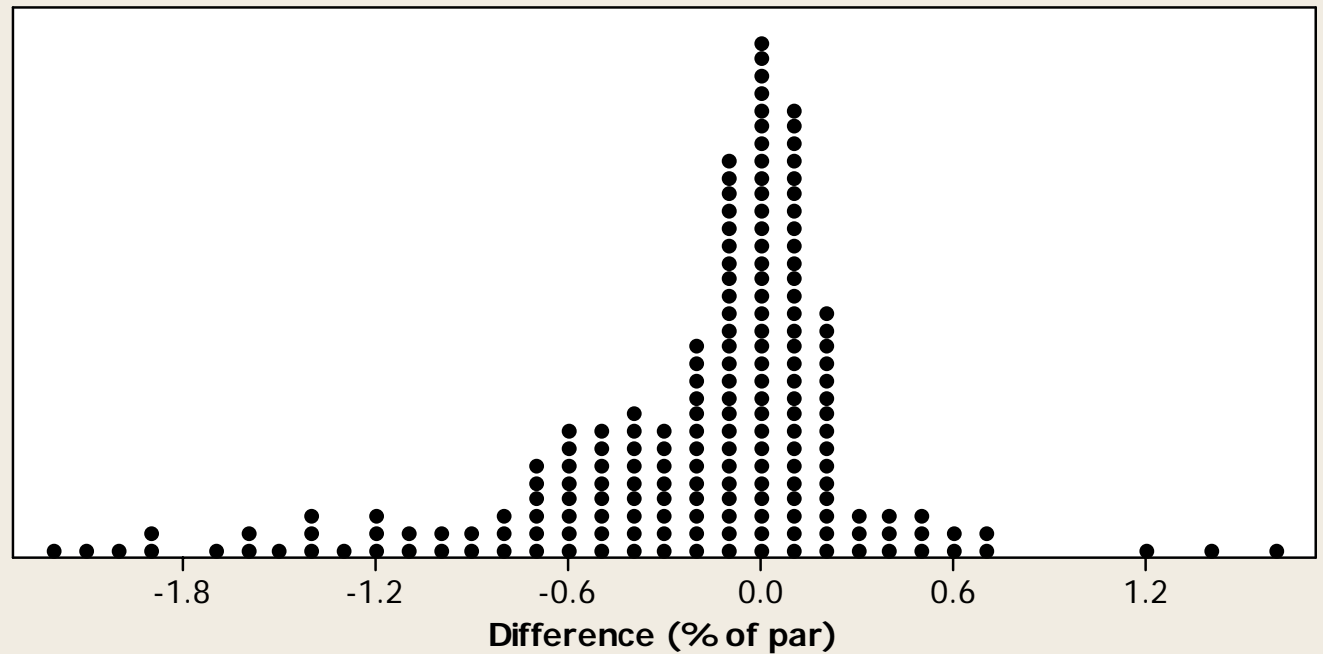
\* *As in paper*

# Distribution of Factors in Sample (363 Fannie and Freddie MBS)



Each symbol represents up to 3 observations.

# Distribution of "Model - Quote"



Each symbol represents up to 2 observations.

# Baseline Inputs Undervalue Recent High-Coupon MBS

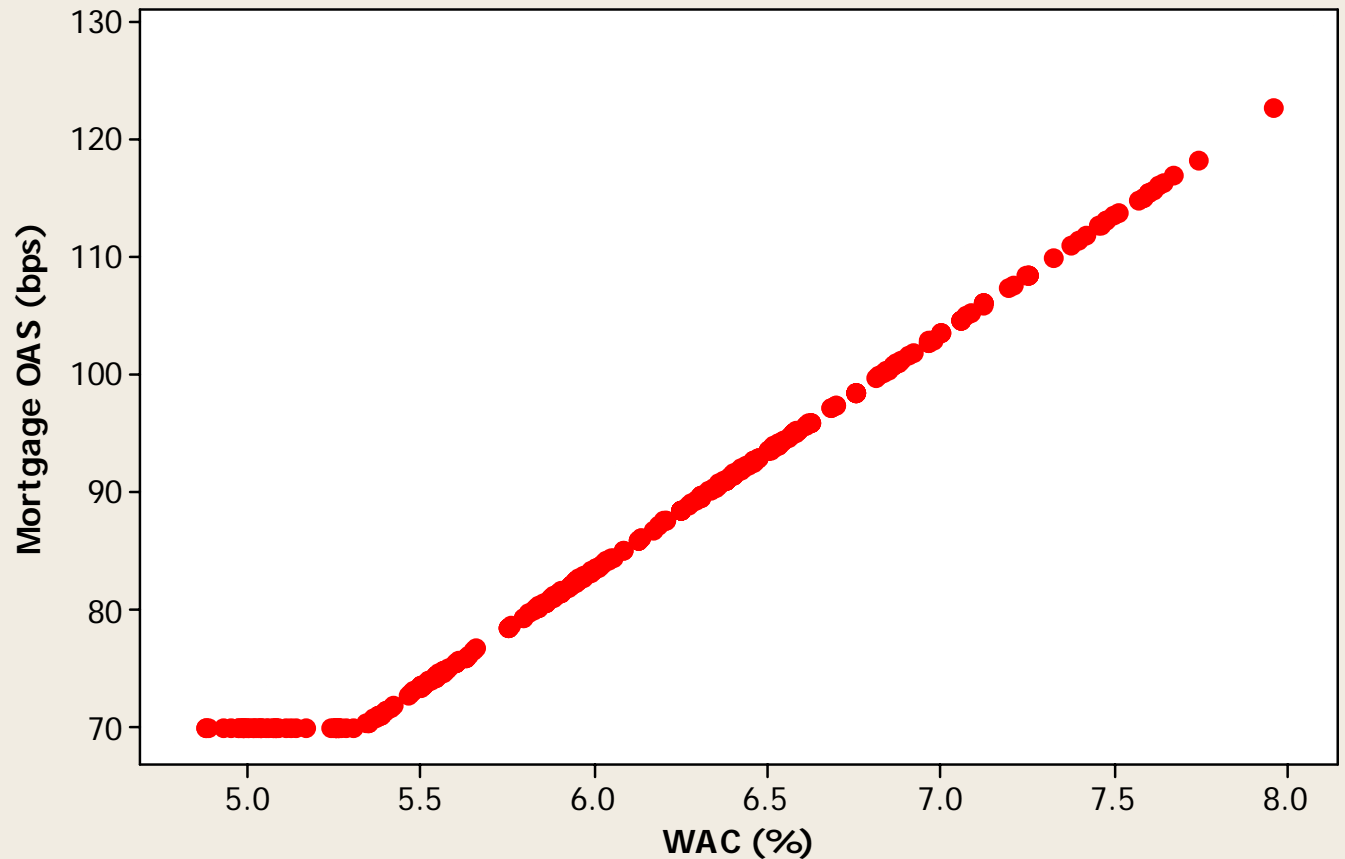
*Above-market WAC* reflects weaker credit,  
which impedes refis

Flat 70 bps OAS overestimates mortgagor's ability  
to refi and thus undervalues MBS

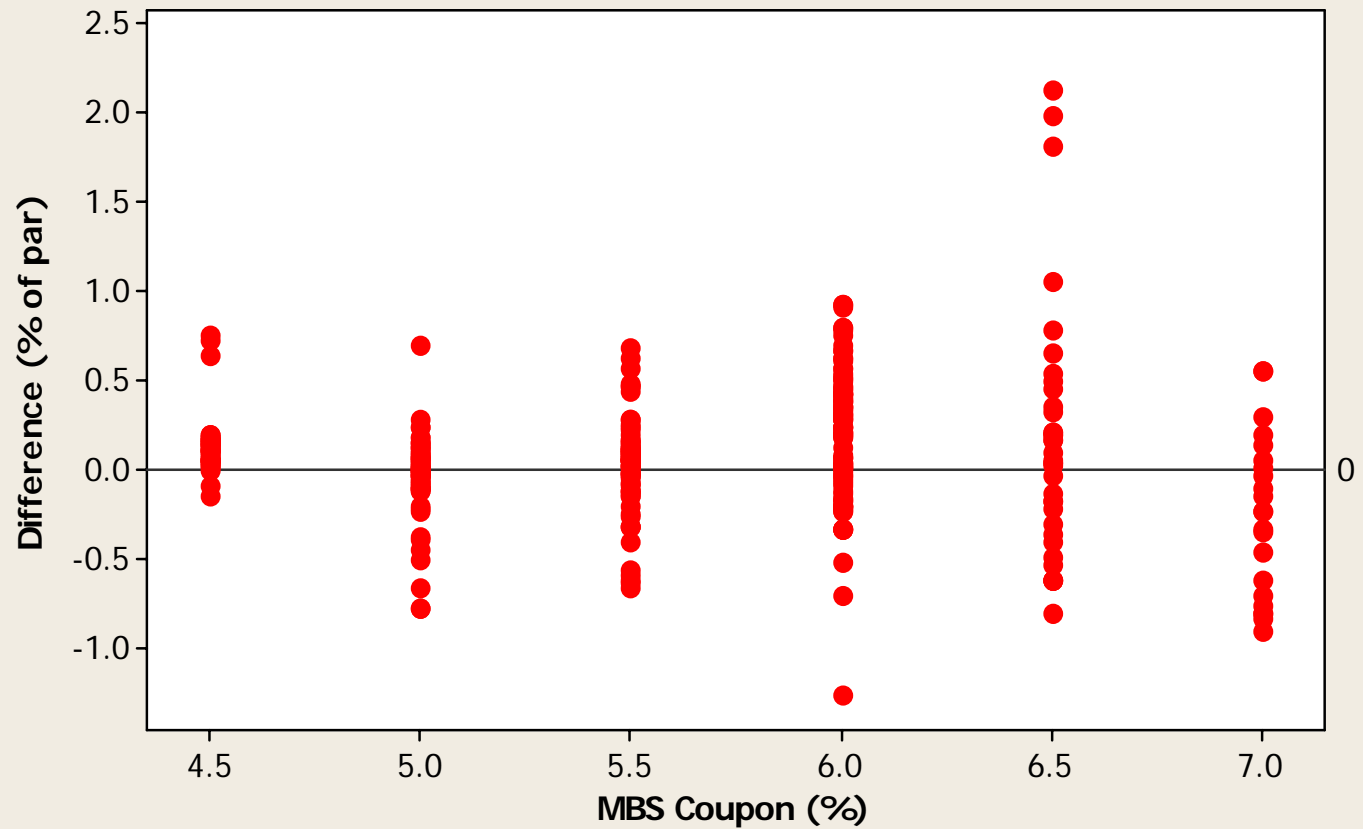
*The cause suggests a cure*

Increase mortgagor OAS to reflect weaker credit  
Slows down refis and increases MBS value

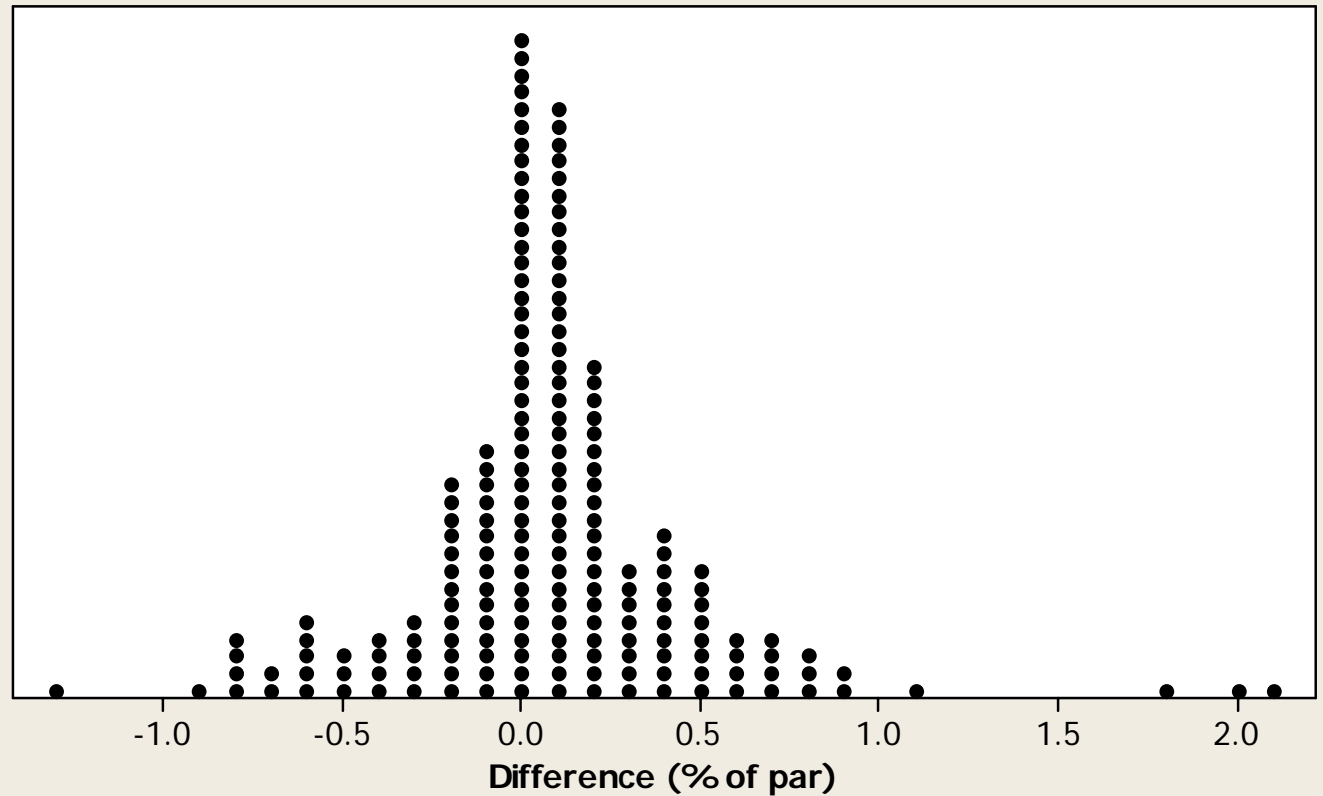
# "Hockey-stick" Adjustment to Mortgagor OAS



# Model Values of High-Coupon MBS Improve

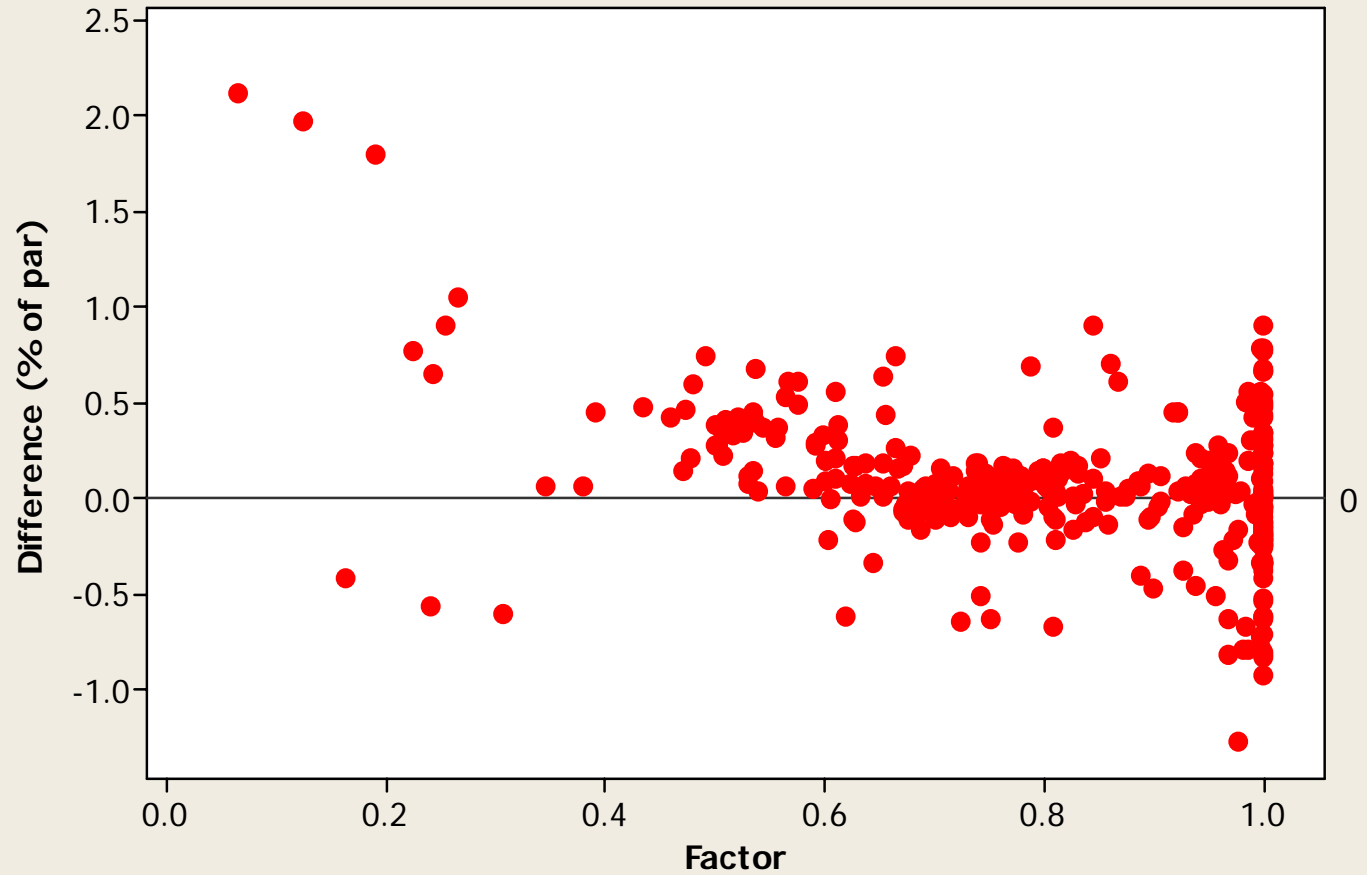


# Resulting in a Much Better Fit

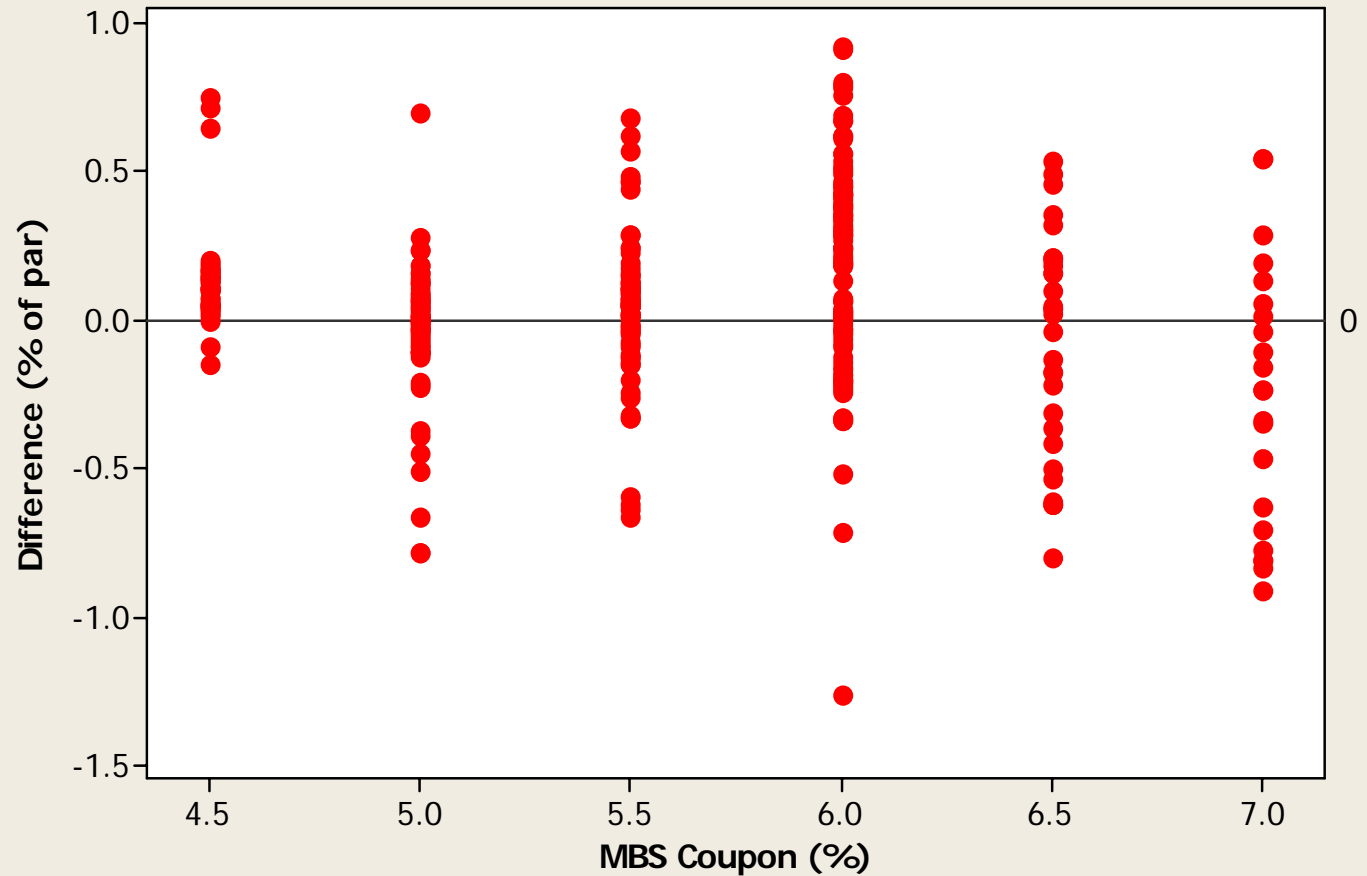


Each symbol represents up to 2 observations.

# But Low-Factor Pools Remain Problematic



# After Removing 9 Low-Factor 6.5% MBS



# Summary Statistics of "Model-Quote" (363 MBS)

| Model Inputs                           | Mean | Median | Std Dev | 1 <sup>st</sup> Quartile | 3 <sup>rd</sup> Quartile |
|--|------|--------|---------|--------------------------|--------------------------|
| Baseline                               | 0.21 | -0.07  | 0.53    | -0.40                    | 0.10                     |
| Hockey-stick adjustment                | 0.07 | 0.06   | 0.37    | -0.09                    | 0.21                     |
| Hockey-stick, without 9 low-factor MBS | 0.05 | 0.05   | 0.32    | -0.09                    | 0.19                     |

*All units % par*

## Calibration of CLEAN™

### Mortgagor OAS should reflect borrower's credit

For Fannie and Freddie pools, 70 bps over swap curve  
(strong BBB credit)

For Ginnie pools, 100bps (weak BBB credit)

### Keep other inputs as in example above

Including laggard distribution and hockey stick  
adjustment

### Estimate MBS OAS from TBA prices

OAS of whole loans should be roughly 30 bps higher

### Refine inputs, depending on precision required

Low factor pools require special handling

Turnover can be customized

# Applications of CLEAN™

## End-of-day pricing

850,000 fixed-rate Agency MBS by NYSE/Sector

## Electronic trading

Indicative pricing for buy-side platform

## Risk management

High-speed stress testing

## Real-time portfolio analysis

## Why is CLEAN™ a Good Model?

Fully consistent with analysis of swaps and bonds

Including volatility-dependent option exercise and interpretation of OAS

Expected prepayment behavior is inferred from the market, rather than history

Just as options trade on implied volatility

User has full control of model

Including parameterization of refinancing behavior

Quick and accurate calculation of prices and sensitivities

10,000 recursive valuations per minute

## References

"Optimum Bond Calling and Refunding", W. M. Boyce and Andrew J. Kalotay, *Interfaces* (November 1979)

"An Option-Theoretic Prepayment Model for Mortgages and Mortgage-Backed Securities", Andrew Kalotay, Deane Yang, and Frank Fabozzi, *International Journal of Theoretical and Applied Finance* (December 2004)

"Is There a Financial Engineer in the House," Andrew Kalotay, *Financial Engineering News* (March/April 2006)

"Refunding Efficiency: A Generalized Approach", Andrew Kalotay, Deane Yang, and Frank Fabozzi, forthcoming in *Applied Financial Economics*

"Optimum Refinancing: Bringing Professional Discipline to Household Finance", Andrew Kalotay, Deane Yang, and Frank Fabozzi, working paper

"A Pointer on Points," Andrew Kalotay and Jinghua Qian, forthcoming in *OR/MS Today*

For Calculators, see [www.kalotay.com/calculators](http://www.kalotay.com/calculators)

CLEAN™ available at [www.kalotay.com/downloads/clean](http://www.kalotay.com/downloads/clean)