

Understanding Returns of Leveraged and Inverse Funds

Examining performance over time

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ABSTRACT

Most leveraged and inverse Exchange Traded Funds (ETFs) are designed to provide a multiple of index returns on a daily basis (e.g., a “fund multiple” of +2x or -2x index returns), before fees and expenses. Because of compounding, a leveraged or inverse ETF’s return over time may be greater or less than that of the index return times the fund multiple. Some reports have suggested that leveraged and inverse ETFs are not appropriate to hold over time, focusing on the effects of compounding on leveraged and inverse funds during a few periods of extraordinary market volatility. This article challenges those assertions by presenting the results from a broader historical analysis of +2x and -2x index returns going back as far as 50 years for the S&P 500 and more than fifteen years for several other indexes based on availability of historical data. In our analyses, we ignored fees, expenses, financing and transaction costs.

Our broad historical analysis encompassing many market conditions showed that, on average, the effect of compounding was almost neutral over time for strategies designed to provide +2x or -2x the daily return of an index. We further show that incidences of achieving returns close to the index return times the fund multiple over time was high for the indexes examined; the shorter the period and the lower the index volatility, the higher the number of incidences. Finally, we show that investors who seek to achieve returns close to the index return times the fund multiple can consider rebalancing. The process involves monitoring the difference between index returns and fund returns and establishing a trigger percentage of deviation as a basis for the rebalancing strategy. A case study of rebalancing is included based on -2x the daily returns of an oil and gas index.

The information contained herein is for educational use only and is not meant to be investment advice. There is no guarantee that any of the strategies mentioned will result in favorable results. Trading will result in commissions and transaction costs, and will create potential tax consequences, which will have a negative impact on the investment returns. The strategies discussed do not take these factors into account. All investing involves risk, including possible loss of principal.

OVERVIEW

Most leveraged and inverse Exchange Traded Funds (ETFs) provide a multiple of index return on a daily basis (e.g., a positive or negative “fund multiple” of +2x or -2x index return). From their launch in June 2006 through June 30, 2009, leveraged and inverse ETFs attracted more than \$33 billion of assets in the U.S.—about 6 percent of all U.S. ETF assets. As of mid-2009, there were 122 leveraged and inverse ETFs, covering a broad range of equity, sector, international, fixed-income, commodity and currency markets.¹ The rapid growth of these types of ETFs has captured the attention of investors and the media—and generated some controversy about where and how this new investment tool best fits in the array of choices available to investors.

What is sometimes forgotten in discussions about these “new” ETFs is that the concept of leveraged and inverse funds is not really new—it is merely the latest chapter in a long history of investment products developed to provide investors access to various indexing strategies. Mutual funds that incorporate leverage or shorting of index exposure have been available in the U.S. for 15 years. Today, there are more than 100 mutual funds that provide leveraged and short exposure to indexes.

Leveraged and inverse mutual funds and ETFs both evolved from unleveraged index fund strategies. Few investors would question the role of index funds in equity portfolio management now, but they, too, generated their share of controversy when they were introduced in the mid-1970s.

Like other index strategies before them, leveraged and inverse ETFs have attracted investors seeking gains from absolute or relative index moves or looking to manage risk.

What, then, is the source of the controversy? Since existing leveraged and inverse funds are designed to achieve a multiple of index return only on a daily basis over time, fund returns can be greater or less than that of the index times the fund multiple. This is a result of the effects of compounding—gains or losses are compounded every day.

Much of the recent discussion of leveraged and inverse funds in the media has centered on how they perform over time. Focusing on a few examples of volatile indexes in a volatile market environment, some have asserted that over time, investors’ returns cannot come close to the fund multiple times the index return, and will always be worse. Some have then concluded that leveraged and inverse ETFs are not appropriate for holding periods greater than one day.

In this article, we take a broader view and provide empirical evidence based on leveraged and inverse S&P 500 returns for a long history (as well as for other indexes) that challenges these assertions. We explain why the funds are designed to provide a

“Some have said [the funds] are downright un-American. Few, if any, investment concepts have been the object of so much scornful ridicule.”

— *Pensions & Investments*,
1975 article on Vanguard’s
first index mutual funds

multiple of index return on a daily basis and show that leveraged funds can be used successfully over time. Rather than focus on narrow examples, we analyzed +2x and -2x leveraged and inverse S&P 500 Index returns for thousands of holding periods ranging from 2 days to 6 months for the past 50 years (excluding fees, expenses, trading and financing costs). These were compared with each period's index return times the fund multiple. We show that:

- On average, the impact of compounding on leveraged and inverse fund returns is virtually neutral for most broad indexes.
- There is a high probability of achieving returns close to the index return times the fund multiple; the shorter the period and the lower the index volatility, the higher the likelihood.
- Rebalancing is an effective tool for investors whose goal is to get close to the index return times the fund multiple over time. The process involves monitoring index returns versus fund returns and establishing a trigger percentage of deviation as a basis for the rebalancing strategy.

Before we present the details and results of our historical study and explain the basic rebalancing process and a case study, we first provide background on the use of the fund multiple over time and highlight some typical investment strategies that involve leveraged and inverse ETFs and mutual funds.

Staying Aligned with the Daily Fund Multiple

Most leveraged and inverse ETFs and mutual funds are designed to provide a multiple of index returns on a daily basis (before fees and expenses). The manager of the fund typically holds stocks, index futures, swaps or short positions along with cash equivalents to pursue this fund multiple on an ongoing basis. To stay aligned with the fund multiple, the fund manager adjusts fund holdings each day based on the closing value of fund assets, reflecting index returns and fund flows for that day.

Day-to-day consistency of index exposure is valuable to many investors over time. Although a leveraged or inverse fund could be created with a longer-term objective, such as a monthly leverage objective, the fund's index exposure would then vary within the month, as gains and losses in between monthly rebalancing change the fund's market exposure. An ETF with a daily leverage objective—say, to be 200 percent exposed to an index—has the objective of providing that same leverage exposure at the end of each and every trading day, regardless of whether an investor bought, held or sold the ETF position on a particular day. Another rationale for adjusting holdings every day to match the fund multiple is that it reduces the risk of the fund experiencing a total loss. (The variation in leverage within the month for a monthly leverage objective fund could be sizable in higher-volatility environments and may lead to a significantly higher degree of leverage than the investor desires.)

Given their daily objectives, it is important to note that leveraged and inverse ETFs have been very successful at delivering returns in line with the fund multiple. In comments on a study of three leveraged and inverse ETFs benchmarked to the Dow Jones Industrial Average, Matt Hougan remarked in an IndexUniverse.com webinar that after examining “over 600 days of trading history... [one-day tracking] was pretty much perfect.”²

Uses of Leveraged and Inverse ETFs for Short and Long Horizons

Leveraged and inverse index exposure in a liquid, transparent ETF can be utilized in a variety of ways, with both short- and longer-term horizons. Since the trading volume for leveraged and inverse ETFs—whether measured in dollars or shares—is several times the percentage of ETF assets, it is likely that leveraged and inverse ETFs are commonly being utilized as short-term tactical trading tools. However, investors also regularly use leveraged and inverse ETFs as a key component of a longer-term portfolio strategy; for example, to pursue returns and manage the risk of long equity and fixed-income positions. The list below identifies a few of the most common applications, all of which can be employed over time:

- Implement a tactical view (long or short) of an index based on an outlook for the economy or segments of the market.
- Overweight or underweight an index exposure, such as a particular market-cap segment, sector or country, by utilizing leverage and thereby avoiding the need to change other positions in the portfolio.
- Hedge or reduce risk, either as a short-term tactical hedge or for longer-term risk management.
- Execute an index-spread strategy designed to capture the relative returns of two indexes. For example, investors may wish to express a view that financial stocks are likely to outperform energy stocks, or that emerging market equities may outperform U.S. large-cap equities.
- Isolate alpha from active strategies. The active risk component of an equity strategy (alpha) can be isolated by hedging the index or beta risk with the benchmark for that strategy using an inverse or leveraged inverse index ETF.

A REVIEW OF HOW COMPOUNDING AFFECTS RETURNS OVER TIME

As previously discussed, leveraged and inverse funds are designed to achieve a multiple of index return only on a daily basis. Over time, returns can be greater or less than that of the index return times the fund multiple. This is a result of the effects of compounding as gains or losses compound daily. Compounding affects all investments over time. It is reflected in index returns, as well as the total returns of unleveraged

mutual funds, stocks and bonds held over time. The magnitude of the compounding effect is related to market conditions that occur during the investment holding period, whether they are upward trending, downward trending or volatile.

Compounding with Unleveraged Investments

In an upward-trending market, compounding can result in longer-term returns that are greater than the sum of the individual daily returns. In **Exhibit 1**, the Index Daily Return column shows that an investment strategy that returns 10 percent a day for two consecutive days generates a 21 percent gain over the two-day period.³ This is greater than the sum of the individual-day returns, or 20 percent. Similarly, in a downward-trending market, compounding can also result in longer-term returns that are less negative than the sum of the individual daily returns. An investment that declines 10 percent a day for two consecutive days would have a -19 percent return, not -20 percent. But in a volatile market scenario, compounding can result in longer-term returns that are less than the sum of the individual daily returns. An investment that rises 10 percent on one day and declines 10 percent the next would have a -1 percent return, which is less than the 0 percent sum of the individual-day returns.

EXHIBIT 1:

Compounding with Unleveraged and Leveraged Investments

Day	Index Daily Return	+2x Fund Daily Return
UPWARD TREND		
1	+10%	+20%
2	+10%	+20%
Compound 2-day Return	+21% [+1%]	+44% [+4%]
DOWNWARD TREND		
1	-10%	-20%
2	-10%	-20%
Compound 2-day Return	-19% [+1%]	-36% [+4%]
VOLATILE MARKET		
1	+10%	+20%
2	-10%	-20%
Compound 2-day Return	-1% [-1%]	-4% [-4%]

Compounding with Leveraged (+2x) Investments: “The Same but More”

Compounding in leveraged funds can result in gains or losses that occur much faster and to a greater degree, as shown in the +2x Fund Daily Return column. In an upward-trending market, compounding can result in longer-term leveraged returns that are greater than two times the return of the unleveraged investment. A leveraged fund that grows 20 percent a day (2 x 10 percent index gain) for two consecutive days would have a 44 percent gain, not two times the 21 percent compound gain of the Index Daily Return. In a downward-trending market, compounding results in +2x leveraged fund returns that are less negative than two times the return of the unleveraged investment. A +2x leveraged fund that declines 20 percent a day (2 x 10 percent index decline) for two consecutive days would have a -36 percent return. This is less negative than two times the 19 percent compound loss of the unleveraged investment.

In a volatile market, compounding can result in leveraged longer-term returns that are less than two times the return of the unleveraged investment. A +2x leveraged fund that rises 20 percent one day (2 x 10 percent index gain) and declines 20 percent the next (2 x 10 percent index decline) generates a -4 percent return. This is a greater loss than the two times -1 percent compound return of the unleveraged investment.

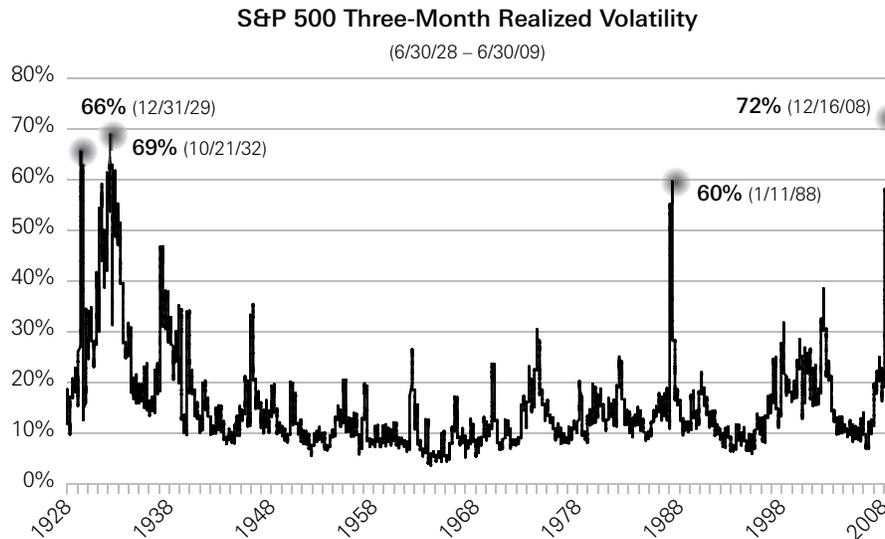
Extreme Volatility in 2008 Magnified the Compounding Effect

Volatility in financial markets reached unprecedented levels in the fall of 2008. The global economy experienced a severe contraction in credit, a loss of confidence in financial institutions, and uncertainty over the type and degree of government intervention. This sparked the most severe economic crisis and financial market volatility since the Great Depression of the 1930s (**Exhibit 2**). As discussed earlier, extremely volatile markets can have a dramatic effect on leveraged and inverse ETFs. Some investors have noticed large differences between the funds' longer-term returns and those (returns) of the index times the fund multiple.

Exhibit 2 shows the annualized volatility for rolling three-month daily returns of the S&P 500 Index from mid-1928 through mid-2009. From mid-September through December 2008, three-month volatility reached its highest level (72%) over this entire time period. (The average volatility over the 81 years was 16%.) Similar extreme volatility levels were reached only two other times over this period: during the Great Depression (1929 to 1932) and in the equity market crash of 1987.

EXHIBIT 2:

Record S&P 500 Volatility Levels Occurred in Late 2008



Source: Bloomberg, three-month rolling volatility based on daily S&P 500 Index returns between 12/31/1927 and 06/30/2009. For illustrative purposes only.

To understand how leveraged and inverse products fit into portfolio strategies, as well as to establish guidelines for their use, it is important to understand how volatility affects returns over time. Specifically, investors should consider the extreme volatility environment that has occurred recently, and the volatility environment that is likely to be experienced over their investment horizons. Volatility depends on a number of economic and financial market factors, but there has already been a decline in U.S. equity risk, with an annualized three-month volatility for the S&P 500 of 34.8% as of June 30, 2009. In addition, the Chicago Board Options Exchange (CBOE) Volatility Index (VIX), a measure of the market's forward-looking view of volatility conditions, was below 26.3% as of the same date.

Analysis of Returns of Leveraged and Inverse Funds Held Over Time

Commentary regarding leveraged and inverse ETFs has frequently cited examples in extremely volatile market conditions to illustrate the effects of compounding on leveraged and inverse fund performance over time. To more completely understand how these ETFs behave over time, it is important to look at a long history of returns for leveraged and inverse index strategies.⁴ To do this, we analyzed strategies designed to provide +2x and -2x the *daily* performance of the S&P 500 Daily Objective Strategies over a 50-year time frame. We performed a similar analysis for the NASDAQ-100 and the Dow Jones financial and energy sector indexes for somewhat shorter time frames based on availability of

historical data. In our analyses, we ignored fees, expenses, financing and transaction costs. We present findings for 2-, 7- and 30-day holding periods on +2x and -2x S&P 500 Daily Objective Strategy returns relating to:

- The average and median difference of returns for +2x and -2x funds compared to the returns of the index times the fund multiple for the same period. Included are selected percentiles from the distribution of these return differences.
- Realized multiples of these +2x and -2x Daily Objective Strategies for 2-, 7-, 30-, 91- and 183-day holding periods versus the S&P 500 Index. These are compared with the fund multiple of +2x or -2x that some investors may want to achieve over time.

Looking at comparative results for this long time frame, we see that the impact of compounding has historically been virtually neutral, with an average effect close to zero and medians close to zero or slightly negative. The overall potential for compounding to lead to positive versus negative effects is approximately equal. We also see a high percentage of periods in which S&P 500 Daily Objective Strategies get close to a +2x or -2x leverage ratio over holding periods of a week and a month. The probabilities of getting close to a +2x or -2x realized multiple falls as the holding period lengthens. Leveraged and inverse Daily Objective Strategy returns for 7- and 30-day holding periods were, at times, the opposite sign to the period target (a “flipped” return), but this was very rare.

Methodology

Our study compares the returns of +2x and -2x S&P 500 Daily Strategies with a period return (defined as +2x or -2x the period index return) for holding periods of 2, 7, 30, 91 and 183 calendar days. Our sample contains all possible two-day, weekly, monthly, quarterly and semiannual holding periods within the past 50 years (1959 through 2008) for the S&P 500 Index. This gives us a large sample to compare all possible end-of-day entry and exit points, but it also leads to overlapping observations. The benefit of this approach is that it removes any potential bias of starting a holding period on a particular day of the week or month.⁵

To focus on the compounding effect, we note some additional assumptions used in this analysis:

- For the +2x and -2x index returns, the leverage ratio is set at the beginning of each period and not changed for the duration of that period. For the +2x and -2x S&P 500 Daily Strategies, the leverage is reset daily to either +2x or -2x. Therefore, the return achieved by the leveraged and inverse Daily Objective Strategies is exactly the daily fund multiple times the daily index return each and every day of the holding periods.
- Index price return is the basis for the analysis.
- All return calculations exclude fees, financing, interest and expenses.
- The +2x and -2x index period returns are not constrained by capital (i.e., losses can exceed -100%).

Analysis of Differences Between +2x and -2x S&P 500 Daily Objective Strategies and Period Index Returns

Exhibit 3 contains statistics from the distribution of differences in return between the Daily Objective Strategy return and +2x and the -2x the index returns for all possible 2-, 7- and 30-day holding periods over the 50-year S&P 500 return history. The averages of the percentage return differences are all essentially zero, and the median is at or just below zero, indicating that leveraged and inverse strategies are about as likely to benefit as to be hurt by the compounding effect for periods up to 30 days for the S&P 500.

Differences between daily target and period leveraged/inverse returns for a long-term S&P 500 history are small.

EXHIBIT 3:

S&P 500 +2x and -2x Daily Strategy vs. +2x and -2x Index Returns

Holding Periods	+2x Differences			-2x Differences		
	2 days	7 days	30 days	2 days	7 days	30 days
Average	0.000%	0.000%	-0.002%	0.000%	-0.002%	-0.035%
97.5th Percentile	0.03%	0.14%	0.71%	0.08%	0.42%	2.08%
75th Percentile	0.00%	0.01%	0.06%	0.00%	0.04%	0.19%
Median	0.00%	0.00%	-0.03%	0.00%	-0.01%	-0.09%
25th Percentile	0.00%	-0.02%	-0.10%	0.00%	-0.05%	-0.29%
2.5th Percentile	-0.02%	-0.12%	-0.50%	-0.07%	-0.35%	-1.54%
% of Periods Positive	53%	43%	39%	53%	43%	39%

Source: Bloomberg, based on daily S&P 500 Index returns for all possible holding periods between 12/31/1958 and 12/31/2008. For illustrative purposes only.

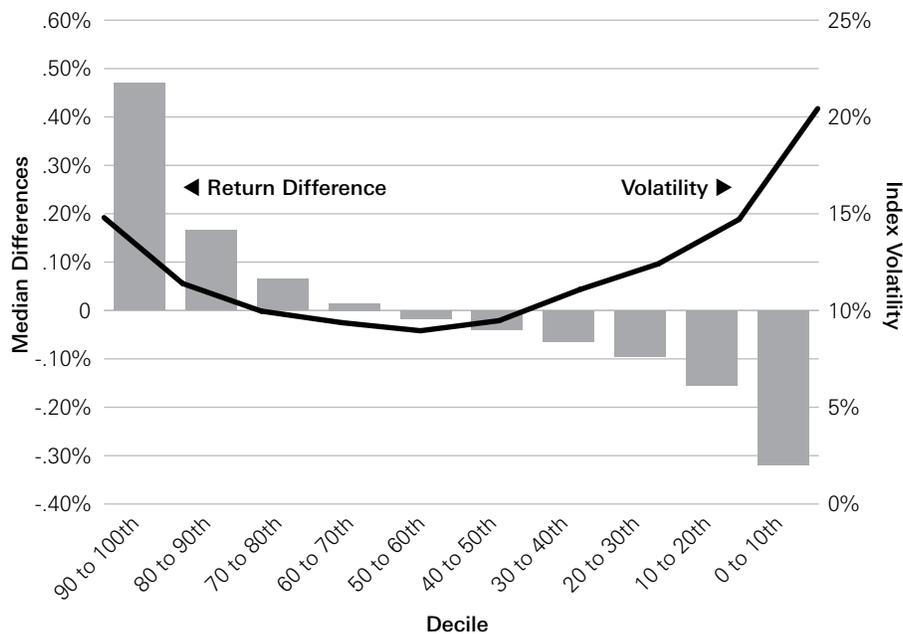
The distribution of the return differences for the +2x and -2x S&P 500 Daily Strategies is tight and balanced over this long history. For example, half of the differences for the +2x Strategy for a 30-day holding period were between 0.1 percent and -0.1 percent. This means that for a +2x Strategy over a 30-day period where the S&P 500 return was 3 percent, the returns were in a range of 5.9 percent to 6.1 percent (compared with 2 x 3 percent, or 6 percent) approximately half the time. In addition, the percentage of positive differences was 53 percent for two-day holding periods. As we move out to longer periods, the return differences are positive about 40 percent of the time.

Volatility Drives the Differences Between Daily Objective Strategy and Index Returns

What drives the size of differences between leveraged and inverse Daily Objective Strategy returns and the +2x or -2x index returns over a longer holding period? As noted above, we would expect the largest driver of compounding effects to be the level of volatility in the market over the investor's holding period. This point is frequently mentioned in academic, analyst and media articles when discussing the performance differences for leveraged and inverse funds held over time. Our long-term study using +2x S&P 500 Daily Strategy returns supports the view that volatility is the key factor driving the size of the differences.

To explore the connection between volatility and variability of returns over the 50-year S&P 500 return history, we first sort the return differences between the 30-day +2x S&P 500 Daily Strategy and the S&P 500 return times two. We then place these return differences into 10 deciles, or buckets, ranking them from the most positive to the most negative. For each of these deciles, we calculate the median return difference and the median annualized 30-day S&P 500 volatility.

EXHIBIT 4:
The Volatility Link: Differences Between 30-Day +2x S&P 500 Daily Strategy Returns vs. +2 Times Index Return



All values are medians for each decile. Source: Bloomberg, based on daily S&P 500 Index returns for all possible 30-day holding periods between 12/31/1958 and 12/31/2008. For illustrative purposes only.

Exhibit 4 displays levels of volatility that are clearly related to the magnitude of the return difference. The holding periods with the most positive and negative deciles of return differences were also ones that have higher volatility. The U-shape of the median volatilities across the return difference deciles reveals that the smallest return differences tend to occur when volatility is lowest. It is notable that the recent episode of extreme volatility was the main factor in observing wider return spreads for longer holding periods for the leveraged and inverse Daily Objective Strategies.

The Impact of Volatility Is Relative to the Magnitude of Index Returns

The degree of impact that volatility has on leveraged Daily Objective Strategy returns is relative to the magnitude of the index return for the period. In periods when index return magnitudes are very large, the return differences tend to be large but positive. In contrast, the periods of the most negative differences (0-10th decile) are those where there have been high S&P 500 volatility levels accompanied by index returns close to zero (that is, flat or trendless markets). In **Exhibit 5**, data from our study of +2x Daily Strategy fund returns show the medians for each decile of return differences, along with the median volatility and absolute value of index return for each bucket. Volatility is not always unwelcome to investors pursuing returns, as higher-magnitude index returns (both positive and negative) are somewhat correlated with higher-volatility market environments.

EXHIBIT 5:

Volatility's Impact on Return Differences Is Affected by the Magnitude of Index Returns

+2x S&P 500 Daily Strategy vs. +2 Times 30-Day Period Index Return

Return Difference Decile	Return Differences*	Absolute Value of S&P 500 Return*	Index Volatility*
90th to 100th	0.47%	6.46%	14.80%
80th to 90th	0.17%	4.63%	11.40%
70th to 80th	0.07%	3.31%	9.97%
60th to 70th	0.02%	2.19%	9.37%
50th to 60th	-0.02%	0.86%	8.95%
40th to 50th	-0.04%	0.32%	9.47%
30th to 40th	-0.06%	0.23%	11.08%
20th to 30th	-0.10%	0.20%	12.41%
10th to 20th	-0.15%	0.09%	14.71%
0 to 10th	-0.32%	0.14%	20.43%

* All values are medians for each decile.

Source: Bloomberg, based on daily S&P 500 Index returns for all possible 30-day holding periods between 12/31/1958 and 12/31/2008. For illustrative purposes only.

Realized Multiples for +2x and -2x S&P 500 Daily Strategies Were Close to Fund Multiple

What is the likelihood that the realized multiple return an investor achieves over a longer holding period is close to the index return times the fund multiple? **Exhibit 6** shows that over relatively short holding periods, there has been a high frequency with which +2x and -2x S&P 500 Daily Strategies got closer to their index return times the fund multiple. Observations of long return histories for more volatile indexes show that the frequencies are lower than for the S&P 500 but still generally high. The results of our study of long-term returns show that the longer the holding period and the more volatile the underlying benchmark, the greater the likelihood that the impact of compounding will cause the returns of a leverage or inverse Daily Objective Strategies to deviate from the fund multiple.

Some fund holders may have an objective of obtaining a realized multiple over time that is close to the fund multiple times the index return. The relevant question to ask is: How close? We calculate the realized multiples for every holding period of 2, 7 and 30 days over the 50 years from 1959 to 2008 to determine the percentage or frequency of realized multiples within selected ranges around +2x and -2x. For leveraged and inverse Daily Strategies that track indexes with volatility profiles similar to or lower than the S&P 500, our analysis indicates that these Daily Strategies have produced realized multiples reasonably close to the +2x or -2x index returns without any rebalancing. **Exhibits 6 and 7** show the results in tabular and graphic form. For a +2x S&P 500 Daily Strategy, as many as 95 percent of the realized multiples fell within a range of +1.5 to +2.5 (compared with a +2x index return) over all possible 30-day holding periods. Even higher percentages result for 2- and 7-day horizons. For a -2x S&P 500 Daily Strategy, somewhat fewer (85 percent) realized multiples fell within a -1.5 to -2.5 multiple range for a 30-day holding period.

EXHIBIT 6:

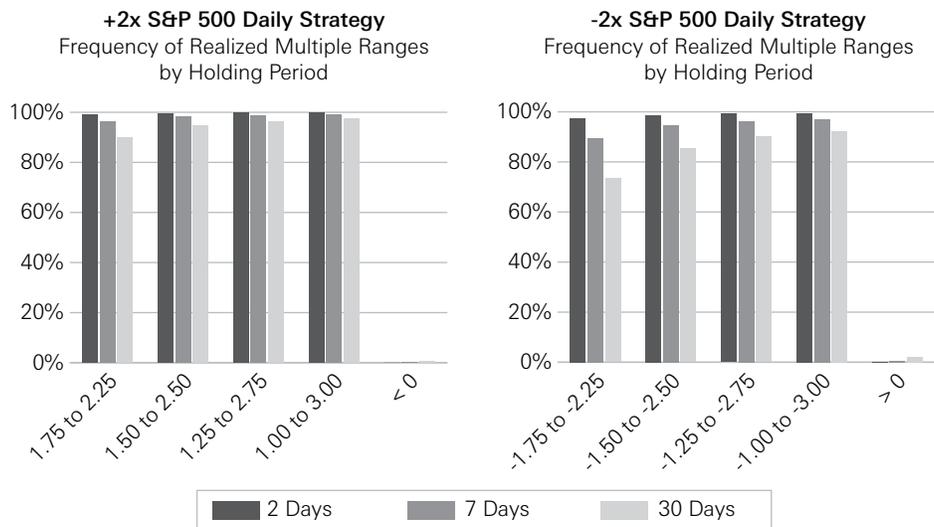
+2x and -2x S&P 500 Daily Strategy Returns:

Frequency of Realized Multiples for 2-, 7- and 30-Day Holding Periods

+2x Strategy				-2x Strategy			
Multiple Ranges	Holding Periods (days)			Multiple Ranges	Holding Periods (days)		
	2	7	30		2	7	30
1.75 to 2.25	99.2%	96.3%	90.0%	-1.75 to -2.25	97.3%	89.5%	73.4%
1.50 to 2.50	99.6%	98.1%	94.7%	-1.50 to -2.50	98.7%	94.6%	85.3%
1.25 to 2.75	99.8%	98.7%	96.5%	-1.25 to -2.75	99.2%	96.3%	90.1%
1.00 to 3.00	99.8%	99.0%	97.3%	-1.00 to -3.00	99.4%	97.1%	92.3%
0.75 to 3.25	99.9%	99.2%	97.8%	-0.75 to -3.25	99.5%	97.7%	93.7%
0.50 to 3.50	99.9%	99.4%	98.2%	-0.50 to -3.50	99.6%	98.1%	94.7%
0.25 to 3.75	99.9%	99.5%	98.5%	-0.25 to -3.75	99.7%	98.4%	95.4%
0.00 to 4.00	100.0%	99.6%	98.7%	0.00 to -4.00	99.9%	99.1%	97.3%
< 0	0.0%	0.2%	0.7%	> 0	0.1%	0.7%	2.0%

Source: Bloomberg, based on daily S&P 500 Index returns for all possible holding periods between 12/31/1958 and 12/31/2008. For illustrative purposes only.

EXHIBIT 7:



Source: Bloomberg, based on daily S&P 500 Index returns for all possible holding periods between 12/31/1958 and 12/31/2008. For illustrative purposes only.

We also observed the frequency of negative multiples for a +2x S&P 500 Daily Strategy and of positive multiples for a -2x S&P 500 Daily Strategy. We refer to these multiples as “flipped.” Flipped multiples happened rarely: less than 1 percent of the 30-day holding periods for a +2x S&P 500 Daily Strategy and about 2 percent for a -2x S&P 500 Daily Strategy.

How does the frequency of the realized multiples change when the holding period is increased to six months? **Exhibit 8** shows the realized multiples for longer-term holding periods, including monthly, quarterly and six-month holding periods for +2x and -2x S&P 500 Daily Strategies. For a -2x Strategy, the frequency at which realized multiples fall within a -1.5 to -2.5 range falls from 85 percent for 30 days to 75 percent for a quarter, and to 70 percent for six months, assuming no rebalancing. The frequency with which returns flip (realized multiples are >0) for the -2x S&P 500 Daily Strategy for a 6-month versus a 30-day holding period rises from 2 percent to 3.4 percent. There is a clear connection between the length of the holding period and the probability of achieving a multiple close to the +2x or -2x index return.

EXHIBIT 8:

**+2x and -2x S&P 500 Daily Strategy vs. +2x and -2x Index:
Frequency of Realized Betas for 30-, 91- and 183-Day Holding Periods**

+2x Strategy				-2x Strategy			
Multiple Ranges	Holding Periods (days)			Multiple Ranges	Holding Periods (days)		
	30	91	183		30	91	183
1.75 to 2.25	90.0%	82.6%	80.0%	-1.75 to -2.25	73.4%	55.4%	44.2%
1.50 to 2.50	94.7%	90.9%	90.1%	-1.50 to -2.50	85.3%	74.9%	70.2%
1.25 to 2.75	96.5%	93.9%	93.5%	-1.25 to -2.75	90.1%	82.6%	79.7%
1.00 to 3.00	97.3%	95.5%	95.1%	-1.00 to -3.00	92.3%	86.6%	84.5%
0.75 to 3.25	97.8%	96.3%	96.1%	-0.75 to -3.25	93.7%	89.4%	88.0%
0.50 to 3.50	98.2%	96.9%	96.8%	-0.50 to -3.50	94.7%	91.0%	90.1%
0.25 to 3.75	98.5%	97.3%	97.3%	-0.25 to -3.75	95.4%	92.2%	91.7%
0.00 to 4.00	98.7%	97.7%	97.7%	0.00 to -4.00	97.3%	95.2%	95.5%
< 0	0.7%	1.1%	1.2%	> 0	2.0%	3.6%	3.4%

Source: Bloomberg, based on daily S&P 500 Index returns for all possible holding periods between 12/31/1958 and 12/31/2008. For illustrative purposes only.

**Realized Multiple Results for Daily Objective Strategies
That Track Higher-Volatility Indexes**

As of mid-2009, more than half of leveraged and inverse fund assets were in ETFs based on the broad-based equity or fixed-income categories. However, many investors also use leveraged and inverse funds tracking U.S. sector indexes with higher return

volatility. To evaluate the realized multiples for Daily Objective Strategies with greater historical risk-reward profiles, we calculate realized multiples over a long-term history for 2-, 7- and 30-day holding periods for Daily Objective Strategies with multiples of +2x and -2x for three other indexes: the NASDAQ-100 Index, the Dow Jones U.S. Financials Index and the Dow Jones U.S. Oil & Gas Index. **Exhibits 9 through 11** show realized multiples for +2x and -2x Daily Objective Strategies based on these indexes. These returns do not illustrate the performance of an actual investment.

The history of daily NASDAQ-100 Index returns begins in 1985, with the index having a return volatility of 28.6 percent over the 1985 to 2008 period. This is significantly higher than the return volatility of 18.3 percent for the S&P 500 over the same period. The frequencies we observe of realized multiples for a +2x and -2x NASDAQ-100 Daily Strategy held for 30 days across all multiple ranges are somewhat lower than the S&P 500 due to the higher volatility of the index, but still above 80%. (The only exception is the -1.75 to -2.25 range for the -2x Strategies.) For example, for a -2x NASDAQ-100 Daily Strategy held 30 days, the -1.50 to -2.50 realized multiple range frequency was 74.1 percent, compared with 85.3 percent for the S&P 500 leveraged strategy.

EXHIBIT 9:

+2x and -2x NASDAQ-100 Daily Strategy:

Frequency of Realized Multiples for 2-, 7- and 30-Day Holding Periods

+2x Strategy				-2x Strategy			
Multiple Ranges	Holding Periods (days)			Multiple Ranges	Holding Periods (days)		
	2	7	30		2	7	30
1.75 to 2.25	97.8%	91.4%	81.1%	-1.75 to -2.25	94.0%	79.1%	57.2%
1.50 to 2.50	98.9%	95.6%	89.4%	-1.50 to -2.50	96.8%	88.4%	74.1%
1.25 to 2.75	99.3%	97.0%	92.9%	-1.25 to -2.75	97.8%	91.5%	80.8%
1.00 to 3.00	99.5%	97.8%	94.7%	-1.00 to -3.00	98.3%	93.4%	84.8%
0.75 to 3.25	99.6%	98.3%	95.7%	-0.75 to -3.25	98.6%	94.6%	87.5%
0.50 to 3.50	99.6%	98.7%	96.4%	-0.50 to -3.50	98.9%	95.6%	89.4%
0.25 to 3.75	99.7%	98.8%	96.9%	-0.25 to -3.75	99.0%	96.1%	90.8%
0.00 to 4.00	99.8%	99.0%	97.2%	0.00 to -4.00	99.3%	97.9%	94.1%
< 0	0.1%	0.6%	1.4%	> 0	0.5%	1.6%	4.4%

Source: Bloomberg, based on daily NASDAQ-100 Index returns for all possible holding periods between 2/05/1985 and 12/31/2008. For illustrative purposes only.

The Dow Jones U.S. Financials and Dow Jones U.S. Oil & Gas index data are available back to 1992, thus providing 17 years of return experience. The annualized return volatilities based on daily data for each index were 24.85 percent and 24.80 percent, respectively; a bit lower than that of the NASDAQ-100 Index, but higher than the S&P 500. The realized multiples for these Daily Objective Strategies are also a bit higher than for the NASDAQ-100 Index, which is precisely what we would expect given the slightly lower return volatilities

of the underlying indexes. Therefore, the analysis of higher-volatility indexes further supports the connection between volatility and holding-period risk for holders of leveraged and inverse funds with daily fund multiples.

EXHIBIT 10:

**+2x and -2x Dow Jones U.S. Oil & Gas Daily Strategy vs. +2x and -2x Index Returns:
Frequency of Realized Multiples for 2-, 7- and 30-Day Holding Periods**

+2x Strategy				-2x Strategy			
Multiple Ranges	Holding Periods (days)			Multiple Ranges	Holding Periods (days)		
	2	7	30		2	7	30
1.75 to 2.25	98.4%	92.6%	83.1%	-1.75 to -2.25	95.1%	80.6%	58.9%
1.50 to 2.50	99.3%	96.2%	91.1%	-1.50 to -2.50	97.6%	89.6%	75.5%
1.25 to 2.75	99.5%	97.2%	93.8%	-1.25 to -2.75	98.4%	92.5%	82.9%
1.00 to 3.00	99.6%	98.0%	95.1%	-1.00 to -3.00	99.0%	94.6%	86.9%
0.75 to 3.25	99.7%	98.5%	96.1%	-0.75 to -3.25	99.2%	95.6%	89.5%
0.50 to 3.50	99.7%	98.7%	96.8%	-0.50 to -3.50	99.3%	96.2%	91.0%
0.25 to 3.75	99.8%	99.1%	97.2%	-0.25 to -3.75	99.3%	96.7%	92.0%
0.00 to 4.00	99.8%	99.1%	97.5%	0.00 to -4.00	99.8%	98.1%	95.1%
< 0	0.1%	0.4%	1.2%	> 0	0.2%	1.4%	3.6%

Source: Bloomberg, based on daily Dow Jones U.S. Oil & Gas Index returns for all possible holding periods between 1/1/1992 and 12/31/2008. For illustrative purposes only.

EXHIBIT 11:

**+2x and -2x Dow Jones U.S. Financials Daily Strategy vs. +2x and -2x Index Returns:
Frequency of Realized Multiples for 2-, 7- and 30-Day Holding Periods**

+2x Strategy				-2x Strategy			
Multiple Ranges	Holding Periods (days)			Multiple Ranges	Holding Periods (days)		
	2	7	30		2	7	30
1.75 to 2.25	98.3%	94.2%	84.5%	-1.75 to -2.25	95.6%	83.6%	64.6%
1.50 to 2.50	99.1%	97.0%	91.3%	-1.50 to -2.50	97.8%	91.1%	78.2%
1.25 to 2.75	99.5%	98.2%	93.9%	-1.25 to -2.75	98.3%	94.1%	84.3%
1.00 to 3.00	99.6%	98.7%	95.4%	-1.00 to -3.00	98.8%	95.4%	87.2%
0.75 to 3.25	99.7%	99.1%	96.2%	-0.75 to -3.25	99.0%	96.1%	89.3%
0.50 to 3.50	99.8%	99.3%	96.8%	-0.50 to -3.50	99.1%	97.0%	91.1%
0.25 to 3.75	99.9%	99.4%	97.2%	-0.25 to -3.75	99.3%	97.4%	92.5%
0.00 to 4.00	99.9%	99.4%	97.6%	0.00 to -4.00	99.7%	98.6%	94.8%
< 0	0.0%	0.3%	1.1%	> 0	0.3%	1.1%	3.9%

Source: Bloomberg, based on daily Dow Jones U.S. Financials Index returns for all possible holding periods between 1/1/1992 and 12/31/2008. For illustrative purposes only.

To summarize these findings, there is a high probability that the realized multiples of the Daily Objective Strategies will be close to the fund multiple over time. The shorter the period and the lower the index volatility, the higher the probability. For longer time periods and more volatile benchmarks, we observed lower probabilities.

FOR LONGER-TERM HOLDING PERIODS AND MORE VOLATILE FUNDS, REBALANCING HELPS CLOSE THE GAP

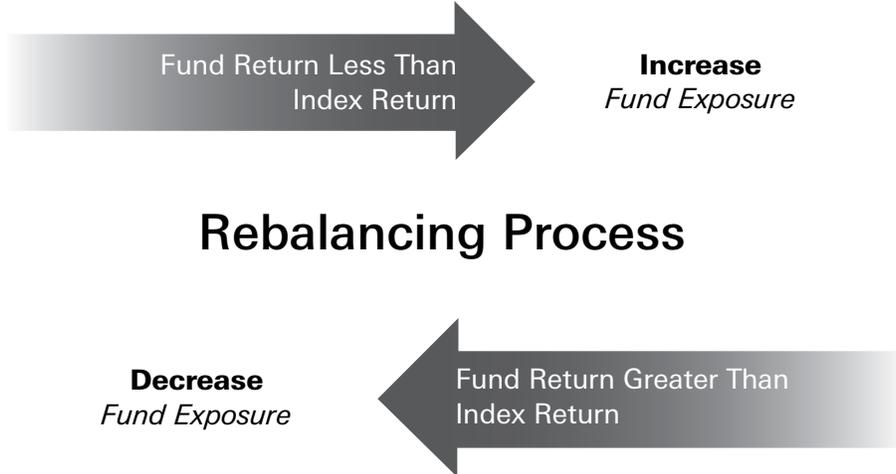
For investors whose goal is to increase the likelihood of getting close to the index return times the fund multiple over time, rebalancing can be an effective strategy.⁶ Investors should routinely monitor the return of the index relative to the performance of the fund and rebalance holdings when they move out of line. The process is analogous to that used by investors rebalancing asset-mix weights versus asset-class policy targets in more conventional investment situations.

It bears repeating that the impact of rebalancing on returns of leveraged and inverse fund strategies is directly related to the effect of compounding. While rebalancing may be helpful if investors seek to match their long-term returns to the fund multiple over time, the returns from such a rebalancing strategy can theoretically be lower than those of an unbalanced strategy in a trending market or low-volatility market environment. Rebalancing has the effect of removing both the negative and potentially positive effects of compounding.

Monitor and Rebalance the Fund Position: Mind the Gap

The rebalancing process for leveraged and inverse fund positions involves carefully monitoring the gap between the index return and fund return, and rebalancing holdings either when this gap moves beyond a specified percentage trigger or at a fixed, calendar-based interval.⁷ **Exhibit 12** shows the investor increasing fund exposure if the fund return is less than the index return, and decreasing fund exposure if the fund return is greater than the index return.

EXHIBIT 12:
Rebalancing Process



For illustrative purposes only.

The size of the rebalance trade for any period can be calculated as the starting fund value multiplied (index return – fund return). **Exhibit 13** demonstrates an extreme case of large daily index returns and rebalancing trades that could be implemented by an investor who wants to increase the likelihood of keeping positions close to the fund multiple over time. Since inverse funds are designed to move in the opposite direction of their underlying indexes, they typically will require a greater frequency and/or degree of rebalancing.

EXHIBIT 13:
Daily Objective Strategy Rebalancing Examples Assume Investor Desires \$200 of Long or Inverse Index Exposure

	Index	+2x Strategy	-2x Strategy
Starting value	\$100	\$100	\$100
One-day return	+10%	+ 20%	- 20%
Ending value	\$110	\$120	\$80
Rebalance trade		\$100* (10% - 20%) = - \$10 (Sell \$10 of Fund)	\$100* (10% - (-20%)) = + \$30 (Buy \$30 of Fund)
Value after rebalance	\$110	\$110	\$110

* Rebalance Amount = Starting Fund Value x (Index Return - Fund Return)

An investor applying the same rebalancing trigger percentage to low- and high-volatility index ETFs with leverage is likely to find that larger-size rebalancing trades may be required, as the potential for larger performance gaps and less proximate realized multiples is other-

wise increased. Alternatively, an investor can set an appropriate trigger for rebalancing by taking into account the volatility of the index and the target rebalancing frequency. For example, to rebalance weekly, the investor could base a trigger on the weekly volatility of the index for the fund. In conclusion, compared with returns from an unbalanced fund, rebalancing fund position(s) reduces the size of the gap such that the investor's realized return from the fund position over multi-day periods is closer to the fund multiple times the index returns.

Case Study of Rebalancing for a -2x Oil & Gas Index Daily Objective Strategy

To demonstrate the potential impact of rebalancing, we looked for a recent historical period with a significantly large difference between an inverse fund return and the fund multiple times the index for a longer holding period. The inverse -2x ETF based on the Dow Jones U.S. Oil & Gas Index has been cited as having large performance gaps for longer periods.⁸ For our rebalancing case study, we evaluated a Daily Objective Strategy based on this index over the period December 2008 through April 2009, when volatility levels were exceptionally elevated and the index rose just 2.2 percent. (For this Daily Strategy, we used index total returns, excluding expenses and trading costs. It does not represent an actual investment or actual ETF returns.)

A -2x Daily Strategy based on this index without rebalancing would have lost -25.8 percent over this period without fees, expenses and transaction costs. This is a difference of 21 percent in return from the -4.4 percent for a -2x period return, given the 2.2 percent index return and excluding fund fees, expenses and transaction costs. How might percentage-trigger rebalancing strategies have fared in narrowing this gap? We analyzed a broad range of percentage-trigger strategies (from 0 percent to 25 percent) and found that any such strategy could have meaningfully reduced the gap in performance. **Exhibit 14** compares the performance for the unbalanced fund with three such sample percentage-trigger rebalancing strategies.

EXHIBIT 14:

**Impact of Rebalancing with a 10% Trigger on Dow Jones U.S. Oil & Gas Index
Holding Period: *December 2008 to April 2009***

Index Return	-2x Index Return	-2x Daily Strategy Return* (Average Days Between Trades)			
		No Rebalancing	Rebalanced: 5% Trigger	Rebalanced: 10% Trigger	Rebalanced: 15% Trigger
2.2%	-4.4%	-25.8%	-2.4% (1.75 Days)	-6.0% (3.81 Days)	-5.2% (6.06 Days)

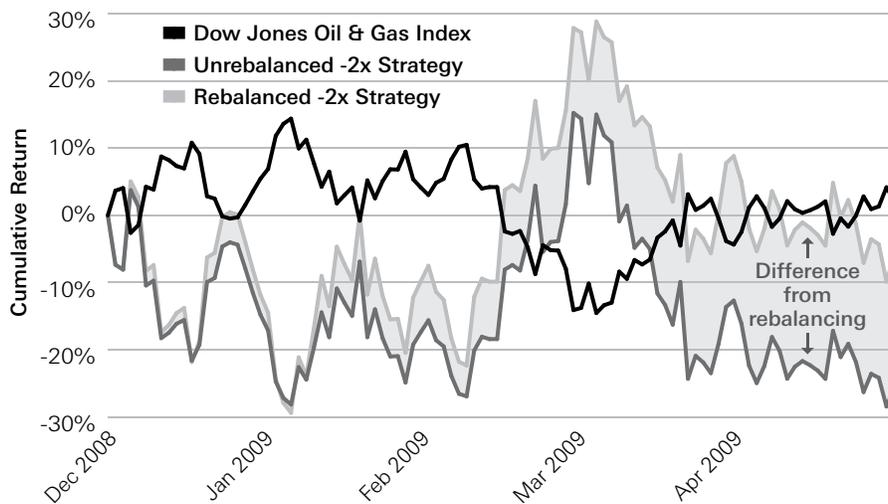
*Excludes trading costs for rebalancing; all rebalancing assumed to be implemented at the close on the same day index return exceeds trigger percentage. For illustrative purposes only.

Given the high level of volatility during the case study period, we focused on the 10 percent gap trigger (that is, we rebalanced when the cumulative difference between index return and fund return was greater than 10 percent) and assumed all rebalancing was done based on end-of-day index levels with no fees, expenses or transaction costs. Using this 10 percent rebalancing trigger, the rebalanced -2x Dow Jones U.S. Oil & Gas Daily Strategy return would have been -6.0 percent, just 1.6 percent below the -4.4 percent return that a -2x fund investor might set as a target for a five-month holding period. There was an average of 3.8 days between rebalances. It is worth noting that in a lower-volatility period, the 10 percent trigger likely would have prompted less frequent rebalancing.

Exhibit 15 shows the index cumulative return over the period for the Dow Jones U.S. Oil & Gas Index (black line), during a very choppy return environment. This result is consistent with the type of periods in which a gap develops for a -2x inverse fund. The dark gray line shows a -2x Daily Strategy with no rebalancing, and the light gray line shows a similar -2x Strategy rebalancing with a 10 percent trigger. This case study is just one example with a specific rebalancing rule employed on a particular index for a single time frame, but it illustrates how a perceived performance gap may be reduced significantly for investors focused on achieving longer-term returns using ETFs designed to provide a multiple of index return on a daily basis.

EXHIBIT 15:

A 10% Rebalancing Band Significantly Reduces Performance Difference for Leveraged Dow Jones U.S. Oil & Gas Index Between December 2008 and April 2009



Cumulative return on Dow Jones U.S. Oil & Gas Index, unrebalanced and rebalanced -2x strategies (using 10% rebalanced band) for period from 12/31/08 through 4/30/09. For illustrative purposes only.

Conclusion

Leveraged and inverse funds have been and can be used successfully over time. Our study shows that the impact of compounding on these funds over multi-day periods for most broad indexes was virtually neutral. There is a high probability of getting close to the index return times the fund multiple over time; the shorter the period and the lower the index volatility, the higher the probability of getting close to the fund multiple times the index return. Finally, rebalancing the size of fund positions, while resulting in some additional trading costs, may be an effective mechanism for investors whose goal is to get closer to the index return times the fund multiple. The process involves monitoring index returns versus fund returns and establishing a trigger percentage of deviation as a basis for the rebalancing strategy.

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References

Marco Avellaneda, Stanley Zhang, “*Path-Dependence of Leveraged ETF Returns*,” Working Paper. Courant Institute of Mathematical Sciences, New York University; Finance Concepts, 2009.

Richard Co, “*Leveraged ETFs vs. Futures: Where Is the Missing Performance?*” CME Group Research & Product Development, February 9, 2009.

Matthew Hougan, “*How Long Can You Hold Leveraged ETFs?*” *Journal of Indexes*, March/April 2009.

Lei Lu, Jun Wang, Ge Zhang, “*Long-Term Performance of Leveraged ETFs*” Working Paper. Shanghai University of Finance and Economics, 2009.

Jason Ruspini, “*Making Leveraged ETFs Work*,” *Seeking Alpha*, July 7, 2009.

Endnotes

¹ Including exchange-traded notes, the total number of leveraged and inverse exchange-traded index products as of June 30, 2009, was 140, with total assets of \$34.7 billion.

² *IndexUniverse.com*, Matt Hougan, “Getting Leverage. Going Short” webinar, May 14, 2009.

³ We use extreme examples, +10 percent and -10 percent, for daily returns for purposes of illustration. These levels of daily returns are highly unlikely for most indexes.

⁴ Over this 50-year daily return history, the annualized volatility of the S&P 500 Index was 15.5 percent.

⁵ Over the 50 years of S&P 500 return history, there were more than 18,000 rolling periods in our sample for each holding period out to six months. The analysis is based on the distribution of differences between the returns of a Daily Objective Strategy for each holding period and the returns of the index for the same period times the fund multiple (2x or -2x). Each holding period return has embedded (but different) compounding effects, allowing us to analyze the distribution of these differences in terms of the arithmetic mean, median and percentiles.

⁶ Some investors in leveraged and inverse funds have strategies based more on a trading view of an index and are primarily seeking to profit from this view. They may be looking to benefit from trending markets or lower levels of short-term volatility and are less concerned about precision in achieving the fund multiple over a longer horizon.

⁷ One weakness of a calendar-based approach is that it does not allow for reaction to large and quick moves that are event-driven, such as were seen in October 1987 and September 11, 2001, and the recent volatility associated with the global financial crisis.

⁸ The Financial Industry Regulatory Authority (FINRA), “Non-Traditional ETFs,” Regulatory Notice 09-31, June 2009.

Important Information

Joanne Hill and George Foster are associated with ProFund and ProShare Advisors LLC.

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