Interest Rate Modeling in the New Era

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Before the credit crunch of 2007, interest rates in the market showed typical textbook behavior. For instance:

**Example 1**
A floating rate bond where LIBOR is set in advance and paid in arrears is worth par (=100) at inception.

\[
100 \cdot \tau_i \cdot L(T_{i-1}, T_i) \quad 100
\]

where \(\tau_i\) is the “length” of the interval \((T_{i-1}, T_i]\), and \(L(T_{i-1}, T_i)\) denotes the LIBOR at \(T_{i-1}\) for maturity \(T_i\).
Example 2
The forward rate implied by two deposits coincides with the corresponding FRA rate: $F_{\text{Depo}} = F_{\text{FRA}}$.

The forward rate implied by the two deposits with maturity $T_1$ and $T_2$ is defined by:

$$F_{\text{Depo}} = \frac{1}{T_2 - T_1} \left[ \frac{P(0, T_1)}{P(0, T_2)} - 1 \right]$$

The corresponding FRA rate is the (unique) value of $K = F_{\text{FRA}}$ for which the following swap(let) has zero value at time $t = 0$.

$$L(T_1, T_2) - K$$
Example 3
Compounding two consecutive 3m forward LIBOR rates yields the corresponding 6m forward LIBOR rate:

\[
(1 + \frac{1}{4} F_{1}^{3m})(1 + \frac{1}{4} F_{2}^{3m}) = 1 + \frac{1}{2} F^{6m}
\]

where

- \( F_{1}^{3m} = F(0; 3m, 6m) \)
- \( F_{2}^{3m} = F(0; 6m, 9m) \)
- \( F^{6m} = F(0; 3m, 9m) \)
Since the credit crunch of 2007, the LIBOR-OIS basis has been neither deterministic nor negligible.
Interest rates in the new era
The explosion of the basis

- Since the credit crunch of 2007, the LIBOR-OIS basis has been neither deterministic nor negligible.

- Likewise, since August 2007 the basis between different tenor LIBORs has been neither deterministic nor negligible.
Interest rates in the new era

The use of different discount and forward curves

- OIS rates are regarded as the best available proxies for risk-free rates.

Example: USD OIS curve as of Sep 24, 2014 ⇒
Interest rates in the new era
The use of different discount and forward curves

- OIS rates are regarded as the best available proxies for risk-free rates. Example: USD OIS curve as of Sep 24, 2014 ⇒

- Banks construct different curves for different LIBOR tenors. Example: USD 3m-LIBOR curve as of Sep 24, 2014 ⇒
Definitions in the multi-curve world

Discount curve

- We assume OIS discounting.
- We consider a tenor $x$ and an associated time structure $T^x = \{0 < T_0, \ldots, T_M\}$, with $T_k - T_{k-1} = x$, $k = 1, \ldots, M$. 
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- We consider a tenor $x$ and an associated time structure $\mathcal{T}^x = \{0 < T_0, \ldots, T_M\}$, with $T_k - T_{k-1} = x$, $k = 1, \ldots, M$.
- OIS forward rates are defined as in the classic single-curve paradigm:

$$F^x_k(t) := F_D(t; T_{k-1}, T_k) = \frac{1}{\tau_k} \left[ \frac{P_D(t, T_{k-1})}{P_D(t, T_k)} - 1 \right]$$

for $k = 1, \ldots, M$, where
  - $\tau_k$ is the year fraction for the interval $(T_{k-1}, T_k]$;
  - $P_D(t, T)$ denotes the discount factor at time $t$ for maturity $T$ for the discount (OIS) curve.
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Consistently with OIS discounting, we assume that risk-adjusted measures are defined by the discount curve.
Definitions in the multi-curve world

Forward LIBOR rates

- The forward LIBOR rate at time $t$ for the period $[T_{k-1}, T_k]$ is denoted by $L^x_k(t)$ and defined by

$$L^x_k(t) = E^T_D [L(T_{k-1}, T_k)| \mathcal{F}_t],$$

where

- $L(T_{k-1}, T_k)$ denotes the LIBOR set at $T_{k-1}$ with maturity $T_k$;
- $E^T_D$ denotes expectation under the (OIS) $T$-forward measure;
- $\mathcal{F}_t$ denotes the “information” available at time $t$. 

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- As in single-curve modeling, $L^x_k(t)$ is the fixed rate to be exchanged at time $T_k$ for $L(T_{k-1}, T_k)$ so that the swaplet has zero value at time $t$:

Value swaplet: 0

\[ L(T_{k-1}, T_k) - L^x_k(t) \]

Time: $t$  $T_{k-1}$  $T_k$
Definitions in the multi-curve world

Forward LIBOR rates

The previous definition of forward LIBOR rate is natural for the following reasons (we omit the superscript $x$):

1. $L_k(t) = E_T D[L(T_{k-1}, T_k) | F_t]$ coincides with the classically defined forward rate in the limit case of a single curve:
   
   $E_T D[L(T_{k-1}, T_k) | F_t] = E_T D[F_D(T_{k-1}; T_k-1, T_k)] = \frac{1}{\tau_k} E_T D[P_F(t, T_{k-1}) - P_F(t, T_k)] P_F(t, T_k)$

2. $L_k(T_{k-1})$ coincides with the LIBOR $L(T_{k-1}, T_k)$:
   
   $L_k(T_{k-1}) = E_T D[L(T_{k-1}, T_k) | T_{k-1}]$

3. $L_k(0)$ can be stripped from market data.

4. $L_k(t)$ is a martingale under the corresponding OIS forward measure.

5. This definition allows for a natural extension of the market formulas for swaps, caps and swaptions.
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$$= \frac{1}{\tau_k} E^T_k\left[\frac{P_D(t, T_{k-1}) - P_D(t, T_k)}{P_D(t, T_k)}\right] = F_D(t; T_{k-1}, T_k)$$

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Definitions in the multi-curve world
LIBOR-OIS basis spreads

Explicit modeling:
- Additive spreads (e.g. M., 2009; Fujii et al., 2009; Amin, 2010)

\[ S^x_k(t) := L^x_k(t) - F^x_k(t), \quad k = 1, \ldots, M. \]

- Multiplicative spreads (e.g. Henrard, 2007, 2009)

\[ 1 + \tau_k S^x_k(t) := \frac{1 + \tau_k L^x_k(t)}{1 + \tau_k F^x_k(t)}, \quad k = 1, \ldots, M. \]

- Instantaneous spreads (e.g. Andersen and Piterbarg, 2010)

\[ P_L(t, T) = P_D(t, T) \int_t^T s(u) \, du \]
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  - Instantaneous spreads (e.g. Andersen and Piterbarg, 2010)
    \[ P_L(t, T) = P_D(t, T) e^{\int_t^T s(u) \, du} \]

- **Implicit modeling:**
  One models the joint evolution of OIS rates and x-curve rates (e.g. M., 2010; Brace, 2010; Kenyon, 2010; Moreni and Pallavicini, 2011; Torrealba, 2011), or risk-free rates and credit events (e.g. Morini, 2012; Filipovic and Trolle, 2012).
The valuation of an interest rate swap (IRS)

- We consider an IRS whose floating leg pays at $T_k$, $k = a, \ldots, b$, the LIBOR with tenor $T_k - T_{k-1} = x$, which is set (in advance) at $T_{k-1}$:

$$\tau_k L(T_{k-1}, T_k)$$

- The time-$t$ value of this payoff is:

$$\tau_k P_D(t, T_k) E^T_{D} \left[ L(T_{k-1}, T_k) | \mathcal{F}_t \right] = \tau_k P_D(t, T_k) L^x_k(t)$$
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- The swap’s fixed leg is assumed to pay the fixed rate $K$ on dates $T'_{c}, \ldots, T'_{d}$, with year fractions $\tau'_j$.

- The IRS value to the fixed-rate payer is given by

$$\text{IRS}(t, K) = \sum_{k=a+1}^{b} \tau_k P_D(t, T_k) L^x_k(t) - K \sum_{j=c+1}^{d} \tau'_j P_D(t, T'_j)$$
The valuation of interest rate swaps

- The corresponding forward swap rate is the fixed rate $K$ that makes the IRS value equal to zero at time $t$:

$$S_{a,b,c,d}(t) = \frac{\sum_{k=a+1}^{b} \tau_k P_D(t, T_k) L_k^x(t)}{\sum_{j=c+1}^{d} \tau_j' P_D(t, T'_j)}$$

<table>
<thead>
<tr>
<th>Swap rate</th>
<th>Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLD</td>
<td>$\frac{\sum_{k=1}^{b} \tau_k P(0, T_k) F_k^x(0)}{\sum_{j=1}^{d} \tau_j' P(0, T'<em>j)} = \frac{1-P(0,T_b)}{\sum</em>{j=1}^{d} \tau_j' P(0, T'_j)}$</td>
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Dual-curve vs single-curve stripping
USD 3m forward rates
The valuation of caplets

Let us consider a caplet paying out at time $T_k$:

$$\tau_k [L(T_{k-1}, T_k) - K]^+$$
The valuation of caplets

- Let us consider a caplet paying out at time $T_k$:
  \[ \tau_k [L(T_{k-1}, T_k) - K]^+ \]

- The caplet price at time $t$ is given by:
  \[
  \text{Cplt}(t, K; T_{k-1}, T_k) = \tau_k P_D(t, T_k) E_D^{T_k} \{ [L(T_{k-1}, T_k) - K]^+ | \mathcal{F}_t \}
  = \tau_k P_D(t, T_k) E_D^{T_k} \{ [L^x(T_{k-1}) - K]^+ | \mathcal{F}_t \}
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$$= \tau_k P_D(t, T_k) E^{T_k}_D \{ [L^x_k(T_{k-1}) - K]^+ | \mathcal{F}_t \}$$

- The rate $L^x_k(t) = E^{T_k}_D [L(T_{k-1}, T_k) | \mathcal{F}_t]$ is, by definition, a martingale under the OIS forward measure $Q^{T_k}_D$. 
The valuation of caplets

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The caplet price at time $t$ is given by:

$$\text{Cplt}(t, K; T_{k-1}, T_k) = \tau_k P_D(t, T_k) E_{T_k}^T \{ [L(T_{k-1}, T_k) - K]^+ | \mathcal{F}_t \}$$

$$= \tau_k P_D(t, T_k) E_{T_k}^T \{ [L^x_k(T_{k-1}) - K]^+ | \mathcal{F}_t \}$$

The rate $L_k^x(t) = E_{T_k}^T [L(T_{k-1}, T_k) | \mathcal{F}_t]$ is, by definition, a martingale under the OIS forward measure $Q_{T_k}^T$.

Let us assume that $L_k^x$ follows a (driftless) geometric Brownian motion under $Q_{T_k}^T$.

Straightforward calculations lead to a (modified) Black formula for caplets.
A payer swaption gives the right to enter at time $T_a = T'_c$ an IRS with payment times for the floating and fixed legs given by $T_{a+1}, \ldots, T_b$ and $T'_{c+1}, \ldots, T'_d$, respectively.
The valuation of European swaptions

- A payer swaption gives the right to enter at time $T_a = T'_c$ an IRS with payment times for the floating and fixed legs given by $T_{a+1}, \ldots, T_b$ and $T'_{c+1}, \ldots, T'_{d}$, respectively.

- Therefore, the swaption payoff at time $T_a = T'_c$ is

$$[S_{a,b,c,d}(T_a) - K]^+ \sum_{j=c+1}^{d} \tau'_j P_D(T'_c, T'_j)$$

where $K$ is the fixed rate and

$$S_{a,b,c,d}(t) = \frac{\sum_{k=a+1}^{b} \tau_k P_D(t, T_k) L_k^x(t)}{C_{D}^{c,d}(t)}$$

$$C_{D}^{c,d}(t) = \sum_{j=c+1}^{d} \tau'_j P_D(t, T'_j)$$
The valuation of swaptions

- The swaption payoff is conveniently priced under the swap measure \( Q_D^{c,d} \), whose associated numeraire is \( C_D^{c,d}(t) \):

\[
\text{PS}(t, K; T_a, \ldots, T_b, T_{c+1}', \ldots, T_d') = \sum_{j=c+1}^{d} \tau_j' P_D(t, T_j') \\
\cdot E_{Q_D}^{c,d} \left\{ \left[ S_{a,b,c,d}(T_a) - K \right]^+ \sum_{j=c+1}^{d} \tau_j' P_D(T_c', T_j') \middle| \mathcal{F}_t \right\} \frac{C_D^{c,d}(T_c')}{C_D^{c,d}(T_c)} \right. \\
= \sum_{j=c+1}^{d} \tau_j' P_D(t, T_j') E_{Q_D}^{c,d} \left\{ \left[ S_{a,b,c,d}(T_a) - K \right]^+ \middle| \mathcal{F}_t \right\}
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The valuation of swaptions

- The swaption payoff is conveniently priced under the swap measure $Q_{D}^{c, d}$, whose associated numeraire is $C_{D}^{c, d}(t)$:

$$PS(t, K; T_a, \ldots, T_b, T'_{c+1}, \ldots, T'_d) = \sum_{j=c+1}^{d} \tau'_j P_D(t, T'_j)$$

$$E_{Q_{D}^{c, d}} \left\{ \frac{[S_{a, b, c, d}(T_a) - K]^{+} \sum_{j=c+1}^{d} \tau'_j P_D(T'_c, T'_j)}{C_{D}^{c, d}(T'_c)} \mid F_t \right\}$$

$$= \sum_{j=c+1}^{d} \tau'_j P_D(t, T'_j) E_{Q_{D}^{c, d}} \left\{ [S_{a, b, c, d}(T_a) - K]^{+} \mid F_t \right\}$$

- Hence, also in a multi-curve set up, pricing a swaption is equivalent to pricing an option on the underlying swap rate.

- Assuming that $S_{a, b, c, d}$ is a lognormal martingale under $Q_{D}^{c, d}$, we obtain a (modified) Black formula for swaptions.
The new market formulas for caps and swaptions

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The new market formulas for caps and swaptions
OIS vs LIBOR discounting

USD Xy10y OIS-based swaption vols

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Pricing general interest rate derivatives

- We just showed how to value swaps, caps, and swaptions under the assumption of distinct discount (OIS) and forward curves.

- What about exotics?

  The pricing of general interest rate derivatives should be consistent with the practice of using OIS discounting. In fact:
  
  - A Bermudan swaption should be more expensive than the underlying European swaptions. In addition, on the last exercise date, a Bermudan swaption becomes a European swaption.
  
  - A one-period ratchet is equal to a caplet.

  Etc ...

- We must forsake the traditional single-curve models and switch to a multi-curve framework.
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  - Etc ...

- We must forsake the traditional single-curve models and switch to a multi-curve framework.
How do we build a multi-curve model?

- Interest-rate multi-curve modeling is based on modeling the joint evolution of a discount (OIS) curve and multiple forward (LIBOR) curves.
How do we build a multi-curve model?

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- Calculating swaption prices in closed form may be hard in general.
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Deterministic basis models

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- Therefore, it makes sense to assume that the LIBOR-OIS basis is constant over time.

- We consider a tenor $x$ and an associated time structure $\mathcal{T} = \{0 < T_0, \ldots, T_M\}$, with $T_k - T_{k-1} = x$, $k = 1, \ldots, M$. 
Deterministic basis models

- In a deterministic and additive basis model one start by modeling OIS rates.

\[
L_x^k(t) = F_x^k(t) + S_x^k(t), \quad k = 1, \ldots, M
\]

where, for \( k = 1, \ldots, M \), the OIS forward rates \( F_x^k \) are defined as in the classic single-curve paradigm, namely:

\[
F_x^k(t) := F_{D}(t; T_{k-1}, T_k) = \tau_k \left[ P_{D}(t, T_{k-1}) - 1 \right]
\]

and the additive basis \( S_x^k \) is deterministic and given by:

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S_x^k(t) = S_x^k(0) = L_x^k(0) - F_x^k(0)
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The pricing of caps is straightforward. The pricing of swaptions is more complex, but presents little difficulty.
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- Do we need to model stochastic basis spreads?

Obviously, we do not have to if, for instance, we price a cap. We clearly have to if we price an option on the LIBOR-OIS basis. But what about non-trivial examples? Bermudan swaptions, CMS spread options, resattable cross-currency swaptions, EUR cash-settled swaptions...
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- Stochastic-basis models can also be introduced for CVA purposes (see next slides).
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- Unilateral CVA is the CVA that focuses on counterparty’s default, and excludes our own.

- Assuming no CSA, unilateral CVA is given by:

\[
CVA = (1 - R)\mathbb{E}[1_{\{\tau \leq T\}}D(0, \tau)V_{\tau}^+]
\]

where
- \( R \) is counterparty’s recovery rate
- \( D(0, t) \) is the discount factor: \( D(0, t) = \exp\{-\int_0^t r(u) \, du\} \)
- \( \tau \) is counterparty’s default time
- \( V_t \) is the portfolio value at time \( t \)
An introduction to CVA

- We repeat the CVA formula:

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\[
CVA = (1 - R) \int_0^T \mathbb{E} \left[ D(0, t) V^+_{t} \right] f_{\tau}(t) \, dt
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where \( f_{\tau} \) denotes the probability density function of \( \tau \).
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- In general, one may want to assume a non-zero correlation between \( \tau \) and market risk factors, thus modeling Wrong-Way Risk (WWR). In this case the CVA formula becomes:
  \[
  \text{CVA} = (1 - R) \int_0^T \mathbb{E} \left[ D(0, t)V_{t}^+ | \tau = t \right] f_\tau(t) \, dt
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CVA of a portfolio of interest-rate deals

- Are interest rates and counterparty’s default independent or correlated?
- To put it differently, what happens to interest rates when a counterparty defaults?
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CVA of a portfolio of interest-rate deals

- Are interest rates and counterparty’s default independent or correlated?
- To put it differently, what happens to interest rates when a counterparty defaults?

- It appears that interest rates jump at default (of a large counterparty).
- But, which rates really jump?
Multi-curve modeling for CVA purposes

- Inspired by historical evidence, M. and Li. (2015) assumed that basis spreads $S^x_k(t)$ jump at counterparty’s default, and that they evolve according to:

$$dS^x_k(t) = J^x_k(t) 1_{\{t \leq \tau\}} (-\lambda dt + dN_t)$$

where $N$ is a Poisson process with constant intensity $\lambda$. 

OIS rates can be assumed to follow any classic single-curve model. LIBORs can then be obtained using:

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For simplicity, we here set $J^x_k(t) \equiv J$, so that (for $t \leq T_{k-1}$):

$$S^x_k(t) = S^x_k(0) + \begin{cases} -\lambda J t & t < \tau \\ -\lambda J \tau + J & t \geq \tau \end{cases}$$

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- We denote by $V_t$ the portfolio’s value at time $t$.

- We assume that $V_t$ is a function of LIBORs $L^x_i(t)$ with $T_i > t$. 

\[ b^x_i(t) := \begin{cases} 
S^x_i(0) - \lambda (T_i - 1) & \text{if } T_i - 1 < t \\
S^x_i(0) + \lambda J t & \text{if } T_i - 1 > t 
\end{cases} \]

which is the time-$t$ basis conditional on default happening at time $t$.

We define $b(t)$ to be the vector of all $b^x_i(t)$'s with ending date $T_i > t$.

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- For any $i = 1, \ldots, M$ and $t \geq 0$, we set:

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Calculating CVA for a portfolio of European-style interest-rate deals

- With some abuse of notation, we write:

\[
V_t = V(t, b(t), F(t))
\]

- The WWR CVA formula is then given by

\[
\text{CVA}_{\text{WWR}} = (1 - R) \int_0^T \mathbb{E}\left[D(0, t) V(t, b(t), F(t))^+\right] \lambda e^{-\lambda t} dt
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- We call independent CVA the CVA obtained by setting \( J = 0 \). Denoting by \( B(t) \) the vector of initial basis spreads \( S_i^x(0) \) with \( T_i > t \), we get

\[ \text{CVA}_{\text{IND}} = (1 - R) \int_0^T \mathbb{E}[D(0, t) V(t, B(t), F(t))^+] \lambda e^{-\lambda t} \, dt \]
Calculating CVA for a portfolio of European-style interest-rate deals

- We derive two CVA approximations by adjusting the initial basis vector.
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- The first approximation is based on neglecting $\lambda J$ terms in vector $b(t)$:

$$b_{i}^{x}(t) \approx \begin{cases} 
S_{i}(0) & \text{if } T_{i-1} < t \\
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- Pretending that the prompt basis jumps as well, we get:

$$\text{CVA}_{\text{WWR}}(B(0)) \approx \text{CVA}_{\text{IND}}(\bar{B}(0))$$

where

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$$

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- So, as a rule of thumb, CVA can be obtained by shifting upwards the initial basis curve in the independent CVA model.
Calculating CVA for a portfolio of European-style interest-rate deals

- The above approximation does not take into account the drift correction coming from the compensated Poisson process.
- A better approximation is based on replacing the time-dependent drift term $\lambda Jt$ with a constant (non-zero) one.
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- To this end, for each basis $S^x_{i+1}(t)$, we define the effective default time $\bar{\tau}_{i+1}$ by

$$
\bar{\tau}_{i+1} := \mathbb{E}[\tau | \tau < T_i] = \frac{e^{\lambda T_i} - 1 - \lambda T_i}{\lambda (e^{\lambda T_i} - 1)} \approx \frac{T_i}{2} - \frac{\lambda T_i^2}{12}
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- Our second approximation then reads as:

$$\text{CVA}_{WWR}(B(0)) \approx \text{CVA}_{IND}(\bar{B}(0) - \lambda J \bar{\tau})$$

where

$$\bar{\tau} := \{\bar{\tau}_1, \ldots, \bar{\tau}_M\}$$
A numerical example: CVA of an interest rate swap

- We consider an ATM 20-year payer interest rate swap (market data as of March 13, 2013).
- Features: the notional is USD 1000, the fixed rate is 2.90%, fixed-leg payments are semi-annual, floating-leg’s quarterly.
- We assume that the instantaneous OIS rate follows a one-factor Hull-White (1990) model:

\[
\text{d} r(t) = \kappa [\vartheta(t) - r(t)] \text{d} t + \sigma \text{d} W(t)
\]

where we set \( \kappa = 0.03 \) and \( \sigma = 0.005 \).

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Table: Exact CVA.
A numerical example: CVA of an interest rate swap

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Table: First CVA approximation. Exact values between ().

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Table: Second CVA approximation. Exact values between ().
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- We have also described the market practice of using OIS discounting in the valuation of the main interest rate derivatives.
- We have shown how to price swaps, caps and swaptions under the assumption of two distinct curves for generating future LIBOR rates and for discounting.
Conclusions

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- Finally, we have introduced a jump-at-default multi-curve model for calculating the CVA of a portfolio of IR deals.