Dear Investor:

MVO has Eaten my Alpha

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Axioma, Inc.

January 28th, 2013
Columbia University
The Mean Variance Optimization Model

Max \( \alpha^T h - \frac{\lambda}{2} h^T Q h \)

st. \( \sum_{i \in U} h_i = 1 \)

\( h \geq 0 \)

Expected Return - Alpha

Risk Model

Risk Term

Holdings
50 Years of Innovation on MVO

• **Expected Return Research:**
  - Separating Alpha from Beta – Risk Premia
  - Integrated backtesting of alpha signals
  - Dynamic weighting of alpha signals, building transparent expected returns

• **Risk Modeling:**
  - Factor models, daily updates, dynamic adaptation to volatility regimes, returns timing
  - Using custom risk models consistent with the alpha process

• **Portfolio construction:**
  - Flexible objectives, multiple risk models, extensive constraint library
  - The role of constraints, the Transfer Coefficient, Constraint Attribution
  - Robust optimization – Dealing with uncertainty in the inputs
  - Alignment issues, the Alpha Alignment Factor
These innovations in MVO were in part motivated by:
- Improving the intuition of optimized portfolios
- Reducing the gap between optimized and “implementable” portfolios
- Correcting for risk underestimation
- But most importantly:

Improving the transfer of information from the alpha signals to realized performance

This is typically reflected by statements of the form:
“We have a good alpha process, but our portfolios are underperforming”
“Our optimized portfolios do not represent our desired bets”
“There is a mismatch between my backtested and realized performance”

In spite of all these improvements, there is still a lot of skepticism around MVO, and we estimate, for example, that only 15% of the AUM in equities is managed by using an MVO model.
The Typical Quant Process: “One Pass” Portfolio Construction – Independence

Alpha Process

Portfolio Construction Strategy

Optimizer

Optimal Portfolio

Risk Process

Very inefficient transfer of information, the optimal portfolio rarely represents the PM’s views present in the Alpha Process.

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Why Should the Portfolio Construction Process be Integrated and Iterated?

- All major sources of systematic risk, whether from vendor models or proprietary alphas, should be captured
  - If not, risk will likely be underestimated and there will be a poor return/risk trade-off

- Constraints may prevent signals of apparent quality delivering strong portfolio performance
  - Measurements of signal quality which ignore risk models and constraints can be highly misleading

- Signal combinations should reflect return/risk trade-offs across the signals
  - These risks and returns depend on constraints and the risk models used
  - Signal weightings should be consistent with portfolio construction implications
An Integrated and Consistent Process

Integrated Alpha, Risk, Construction Strategy

MVO (what alpha?, what risk model?)

Optimal Portfolio

Goal of a Consistent Process

- Signals of high, transferable, quality
- An optimal portfolio which is an accurate representation of the signals
- A high transfer of information
- Accurate risk estimation
Risk Model Notation - $Q$

- **Terminology and Notation**
  - $X_A$ is an array of alpha factors
  - $X_R$ is an array of *other* factors (risk factors)
  - $X = [X_A, X_R]$ is the total factor array
  - $\Omega = \begin{bmatrix} \Omega_A & \Omega_{A,R} \\ \Omega_{R,A} & \Omega_R \end{bmatrix}$ is the factor covariance matrix
  - $\Delta^2$ is a diagonal matrix of specific risks

- The risk model is the matrix, $Q$, given by

\[ Q = X^T \Omega X + \Delta^2 \]
Use Theory as Basis for Consistency

- IC is proportionally related to performance (Fundamental Law). So, improvement in IC should directly translate to improved performance.

- Turn signals into portfolios by neutralizing against a set of factors

- These portfolios are called factor-mimicking portfolios (FMPs)

- FMPs will help us measure signal quality

- We will combine FMPs into “target portfolios” (our views)

- We will derive an implied alpha from the target portfolios

- We’ll use this “alpha” in the MVO model

We will demonstrate that the theory works in practice if we really follow a consistent process.
Factor-Mimicking Portfolios

Assume we have \( m \) signals. For each signal \( j, 1 \leq j \leq m \), the FMP is the solution, \( h_j \), to

\[
\begin{align*}
\text{minimize} & \quad h^T Q h \\
\text{subject to} & \quad X_R^T h = 0 \\
& \quad X_A^T h = e_j
\end{align*}
\]

- The return and risk of \( h_j \) provide measures of the **transferable** quality of signal \( j \)
- We can compute the IC of the signal by looking directly at the FMP for that signal
- FMPs implement our “views” with respect to our signals
FMPs and MVO

If we have a portfolio (FMP), and we believe in MVO, how do we make sure that MVO gives us back the FMP?

If we set
\[ \tilde{\alpha} = Qh_j \]

and solve the MVO problem

\[
\begin{align*}
\text{minimize} & \quad \tilde{\alpha} - \lambda h^T Qh \\
\text{subject to} & \quad X_R^T h = 0
\end{align*}
\]

we get back a multiple of \( h_j \) as the solution
Now to Multiple Signals

- Given a set of weights, $\omega_1, \ldots, \omega_m$, we call
  \[ h^* = \sum_j \omega_j h_j \]
  the target portfolio

- As before, given a target portfolio, $h^*$, let's consider the corresponding implied alpha, $\tilde{\alpha} = Qh^*$

- When we use this alpha in our MVO problem,
  \[
  \text{minimize} \quad \tilde{\alpha} - \lambda h^T Qh \\
  \text{subject to} \quad X^T_K h = 0
  \]
  we get back a multiple of the target portfolio, $h^*$
One Approach to Weighting Signals

- Given a time series of FMPs for each alpha signal, we can estimate:
  
  - $E[f_A]$ (Expected Returns of FMPs)
  - $\Omega_A$ (Estimated Covariance of FMP Returns)

- Determine FMP weights, $\omega$, by solving the optimization problem:
  
  $$\omega = \arg \max E[f_A]^T v - \lambda v^T \Omega_A v$$

- Use these weights to determine the target portfolio $h^*$

- In practice a portfolio manager adjust the estimate of $E[f_A]$ over time to do factors timing.

- Alternatively, the manager can just set $E[f_A]$ to a vector of ones which implies that taking a min variance position in FMPs with consistent risk premiums.
Measuring the Efficiency of Signal Transfer

- The transfer coefficient

\[ TC = \frac{\alpha^T w}{\sqrt{\alpha^T Q^{-1} \alpha} \sqrt{w^T Q w}} \]

measures how well a signal is transferred to the final portfolio \( w \). It can be viewed as the correlation between the risk-adjusted alpha, \( Q^{-1/2} \alpha \), and the risk-weighted portfolio, \( Q^{1/2} w \).

- We introduce another measure

\[ TC^* = \frac{w^T Q h^*}{\sqrt{(h^*)^T Q (h^*)} \sqrt{w^T Q w}} \]

to indicate the extent of transfer from the target portfolio \( h^* \) to the final portfolio \( w \).

- Observe that, when we set \( \alpha = Q h^* \), the two definitions agree.
Dollar-Neutral Example

- Three alpha signals:
  - Momentum, ROE, Sales/Price

- We computed a covariance matrix, $\Omega_A$, of the corresponding FMP returns

- We chose a set of factor weights, $\omega$, as the solution of the problem
  - Minimize $v^T \Omega_A v$
  - Subject to $\sum v_i = 1, v \geq 0$
Comparing Different Quant Processes

• We considered two different risk models:
  – WW21AxiomaMH and WWAllAlphas
  – WW21AxiomaMH is the “standard” Axioma Risk Model, with three different definitions of MTM, Growth and Value than the definitions in the alpha signals
  – WWAllAlphas replaces MTM, Growth, and Value with three alpha signals

• We considered two strategies:
  – Unconstrained and Constrained (neutral to all “risk factors”)

• We considered three different values for alpha:
  – Ideal Uses consistent process we outlined
  – Raw Weights raw alpha signals using $\omega$
  – Ortho Weights orthogonalized alpha signals (orthogonal to risk factors) using $\omega$
How Well is the Information Transferred?

Transfer Coefficient for Constrained Strategy

<table>
<thead>
<tr>
<th></th>
<th>WWAllAlphas</th>
<th>WW21AxiomaMH</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Alpha</strong></td>
<td>TC</td>
<td>TC*</td>
</tr>
<tr>
<td><strong>Ideal</strong></td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Raw</strong></td>
<td>0.77</td>
<td>0.93</td>
</tr>
<tr>
<td><strong>Orthogonal</strong></td>
<td>0.74</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Transfer Coefficient for Unconstrained Strategy

<table>
<thead>
<tr>
<th></th>
<th>WWAllAlphas</th>
<th>WW21AxiomaMH</th>
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</thead>
<tbody>
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<td>TC</td>
<td>TC*</td>
</tr>
<tr>
<td><strong>Ideal</strong></td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Raw</strong></td>
<td>1.00</td>
<td>0.72</td>
</tr>
<tr>
<td><strong>Orthogonal</strong></td>
<td>1.00</td>
<td>0.71</td>
</tr>
</tbody>
</table>
## Optimal Portfolio Characteristics

<table>
<thead>
<tr>
<th>Alpha</th>
<th>Risk Model</th>
<th>Pure Alpha Portfolio Weights (normalized)</th>
<th>Portfolio R-Squared</th>
<th>Risk Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ROE</td>
<td>Sales-to-Price</td>
<td>Momentum</td>
</tr>
<tr>
<td>h*</td>
<td></td>
<td>0.69</td>
<td>0.22</td>
<td>0.09</td>
</tr>
<tr>
<td>Ideal</td>
<td>WWAllAlphas</td>
<td>0.69</td>
<td>0.22</td>
<td>0.09</td>
</tr>
<tr>
<td>Raw</td>
<td>WWAllAlphas</td>
<td>0.69</td>
<td>0.11</td>
<td>0.01</td>
</tr>
<tr>
<td>Ortho</td>
<td>WWAllAlphas</td>
<td>0.69</td>
<td>0.17</td>
<td>0.01</td>
</tr>
<tr>
<td>Ideal</td>
<td>WW21AxiomaMH</td>
<td>0.69</td>
<td>0.30</td>
<td>0.11</td>
</tr>
<tr>
<td>Raw</td>
<td>WW21AxiomaMH</td>
<td>0.69</td>
<td>0.43</td>
<td>-0.02</td>
</tr>
<tr>
<td>Ortho</td>
<td>WW21AxiomaMH</td>
<td>0.69</td>
<td>0.81</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

- Results of regressing optimal portfolio against individual alpha representing FMPs.

- It is also interesting to look at exposures of the optimal portfolios to raw alpha signals and the contribution to variance of the alpha factors as determined by the WWAllAlphas model.
Long-only Example – Full backtest

- Long Only Case (Benchmark & Universe: FTSE All-World)
  - Maximize alpha (implied alpha of $h^*$) Subject To:
  - Long-Only and Fully Invested
  - Round-Trip (two-way) Turnover restricted to be at most 20%/month.
  - Active Predicted Beta Bounds of ± 0.02
  - Active Asset Bounds of ± 3%
  - Active Industry and Country Bounds of ± 3%
  - Maximum Predicted Active Risk of 3%
  - Style Exposure Bounds of ± 0.1 on Liquidity, Leverage, STM, Size, Exchange-Rate Sensitivity, and Volatility

- Backtest Details
  - Portfolio rebalanced at the end of each month
  - Backtest Period: 2000-12-29 to 2012-08-31
  - Consistent Approach: Custom risk model built using Axioma’s RMM and including all the factors that were used to construct the alpha (WWAllAlphas)
Transferring Information with a Consistent Process

<table>
<thead>
<tr>
<th></th>
<th>WW21AxiomaMH</th>
<th>WWAllAlphas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized Active Return</td>
<td>2.66%</td>
<td>2.84%</td>
</tr>
<tr>
<td>Annualized Active Risk</td>
<td>3.37%</td>
<td>2.96%</td>
</tr>
<tr>
<td>IR</td>
<td>0.79</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Comparison of Active Returns in Backtests

- Using consistent process provided more return and less risk.
- Returns between the two models are highly correlated, but the slightly higher average active return and considerable less volatile active returns adds up over time.
A Consistent Approach Improves the Representation of the Alpha Factors

Alpha Factor Exposures of Optimized Portfolio When Using A Consistent Approach

Alpha Factor Exposures of Optimized Portfolio using WW21AxiomaMH

[Diagrams showing time series plots for different factors and weights]
A Consistent Approach Improves Implementation Efficiency

- The proportions of exposures of the alpha factors in the two backtests may look similar, but are they?

- The time-series of cross-sectional correlations between alpha factor exposures and FMP weights reveal a different story
  - Average correlations are 83% for Consistent Approach and 67% for WW21AxiomaMH

- This occurs due to both constraints used in the strategy and the relative risks of the alpha factors not being captured properly, i.e., misalignment in the risk model and constraints.

- Again, this case is not that extreme because even the base model is relatively “aligned” to the custom risk model.

- In practice, if there is more misalignment, or the strategy is more heavily constrained, we would see a much bigger difference.
What Would Have Happened With a Naïve Approach (Raw) to Signal Weighting?

- Run the same strategy except that the expected return was created by weighting the three raw alpha factors (not FMPs) by their naïve Ics.

<table>
<thead>
<tr>
<th>Average Naïve ICs</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1Y Momentum:</td>
<td>0.025</td>
</tr>
<tr>
<td>ROE:</td>
<td>0.021</td>
</tr>
<tr>
<td>Sales/Price:</td>
<td>0.011</td>
</tr>
</tbody>
</table>

- The realized performance with these naïve construction of the expected returns was worse

<table>
<thead>
<tr>
<th></th>
<th>Alpha</th>
<th>Alpha Raw</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized Active Return</td>
<td>2.84%</td>
<td>2.53%</td>
</tr>
<tr>
<td>Annualized Active Risk</td>
<td>2.96%</td>
<td>2.87%</td>
</tr>
<tr>
<td>IR</td>
<td>0.96</td>
<td>0.88</td>
</tr>
</tbody>
</table>

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The Alpha Signals are Better Represented with the Consistent Approach

[Graphs showing Alpha Factor Exposures of the Optimized Portfolio]
Summary

• An integrated investment process can greatly enhance the efficiency of information transfer from signals to portfolios

• This process should ensure that all known sources of risk are adequately captured and included in the optimizations

• Assessments of signal quality will then be more relevant to final portfolio returns and risk budget allocation

• We introduced a new measure of transfer, $T_C^*$, that measures how accurately you are tracking $h^*$

• The alpha used as an “expected return” in the final MVO computation will generally differ from direct combinations of raw signals. This alpha will depend on constraints and the risk models.
References


