The science and art of nonlinear valuation: from risk neutral to indifference XVAs

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Joint work with co-authors listed in the references
Old world specimen: Sticky Ratchet Knockout Cap

And: **No credit – margin– funding – capital, only basic asset class**
Old world specimen: Sticky Ratchet Knockout Cap

7 Sept- 8 Oct 2008: Fannie Mae, Freddie Mac, Washington Mutual, Landsbanki, Glitnr, Kaupthing, Lehman Brothers (Merril Lynch)
Old world specimen: Sticky Ratchet Knockout Cap

We don't get around much anymore
New world specimen: hello again IRS, but with....

Complex payoff, simple system \(\rightarrow\) Simple payoffs, complex system
New world specimen: hello again IRS, but with....

Complex payoff, simple system $\rightarrow$ Simple payoffs, complex system

Let’s build this complex risk/cost system one step at the time.
Basic payout in the old Quants pocket universe

\[ V_t = E_t \left[ \Pi(t, T^{\wedge} \tau) + \ldots \right] \]  
\( \tau \) is the first default time between bank and counterparty

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Nonlinear valuation & XVA
Basic payout in the old Quants pocket universe

Derivatives notional 2011 (BIS) USD 708 trillions \(7.08 \times 10^{14}\). World GDP 2011: 79 Trillions. Start from derivative’s basic cash flows

\[ V_t := \mathbb{E}_t[ \Pi(t, T \wedge \tau) + \ldots ] \quad (V = \text{Expected[DiscountedCashFlows]}) \]

\(\tau\) is the first default time between bank and counterparty.
Including credit, collateral, funding & multi-curve effects

Updating cash flows to include all effects

Adding pre-default Collateral Flows
Including credit, collateral, funding & multi-curve effects

Updating cash flows to include all effects

Basic Payout with Collateral Costs & Benefits

• Collateralization procedure cash flows.

\[ V_t := \mathbb{E}_t[ \prod(t, T \land \tau) + \gamma(t, T \land \tau; C) + \ldots ] \]

where

\[ \rightarrow C_t \] is the collateral account defined by the CSA,
\[ \rightarrow \gamma \] is the collateral margining cost (of carry).
Basic Payout with Collateral Costs & Benefits

- Collateralization procedure cash flows.

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where

- \( \rightarrow \) \( C_t \) is the collateral account defined by the CSA,
- \( \rightarrow \) \( \gamma \) is the collateral margining cost (of carry).

\[ \gamma \approx - \sum_{k=1}^{n-1} 1\{t \leq t_k < \tau \land T\} D(t, t_k) C_{tk} (t_{k+1} - t_k) (\tilde{c}_{tk} - r_{tk}) \]

\[ \tilde{c} = c^+ \text{ if } C > 0, \quad \tilde{c} = c^- \text{ if } C < 0. \]

Note that if the collateral rates in \( \tilde{c} \) are both equal to the risk free rate, then this term is zero.

- The second expected value: ColVA/VMVA
A Little History: CVA and DVA can be sizeable.

**Citi, 1Q 2009**: “Revenues also included [...] a net 2.5$ billion positive Credit Valuation Adjustment (CVA) on derivative positions [...];

**BIS**: In crisis, roughly 2/3 of losses attributed to counterparty credit risk were due to CVA losses and only about 1/3 to actual defaults.

**B. et al [24]** (2012), under strong contagion at counterparty default (eg CDS) collateral can leave a sizeable CVA [Initial Margin?]

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Nonlinear valuation & XVA

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Close-Out: Trading-CVA/DVA after Collateralization

- Third contribution: cash flow at 1st default

\[ V_t := \mathbb{E}_t \left[ \Pi + \gamma + 1_{\{\tau<T\}} D(t, \tau) \theta_\tau(C, \varepsilon) + \ldots \right] \]

where \( \varepsilon_\tau \) is the close-out amount, or residual value of the deal at default, “exposure at default”. We can include liquidation delays.
Including credit, collateral, funding & multi-curve effects  

Updating cash flows to include all effects

Close-Out: Trading-CVA/DVA after Collateralization

- Third contribution: cash flow at 1st default

\[ V_t := \mathbb{E}_t \left[ \Pi + \gamma + 1_{\{\tau < T\}} D(t, \tau) \theta_T(C, \varepsilon) + \ldots \right] \]

where \( \varepsilon_T \) is the close-out amount, or residual value of the deal at default, “exposure at default”. We can include liquidation delays.

- Replacement closeout, \( \varepsilon_T = V_T \Rightarrow \) nonlinearity, difficult!

\[ V_t := \mathbb{E}_t \left[ \Pi + \gamma + 1_{\{\tau < T\}} D(t, \tau) \theta_T(C, V_T) + \ldots \right] \]
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$$V_t := \mathbb{E}_t \left[ \Pi + \gamma + 1_{\{\tau < T\}} D(t, \tau) \theta_\tau(C, V_\tau) + \ldots \right]$$

- Default cash flow $\theta_\tau$ calculated by ISDA documentation

$$\theta_\tau(C, \varepsilon) := \varepsilon_\tau - 1_{\{\tau = \tau_C < \tau_I\}} \Pi_{CVAcoll} + 1_{\{\tau = \tau_I < \tau_C\}} \Pi_{DVAcoll}$$

For example, in case of re-hypothecation (same LGD),

$$\Pi_{DVAcoll} = \text{LGD}_I(\varepsilon_\tau - C_\tau - \Pi_{CVAcoll} + 1_{\{\tau = \tau_I < \tau_C\}} \Pi_{DVAcoll}) \text{.}$$
Close-Out: Trading-CVA/DVA after Collateralization

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\[ V_t := \mathbb{E}_t \left[ \Pi + \gamma + 1_{\{\tau < T\}} D(t, \tau) \theta_{\tau}(C, \varepsilon) + \ldots \right] \]

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\[ \theta_{\tau}(C, \varepsilon) := \varepsilon_{\tau} - 1_{\{\tau = \tau_C < \tau_I\}} \Pi_{\text{CVAcoll}} + 1_{\{\tau = \tau_I < \tau_C\}} \Pi_{\text{DVAcoll}} \]

- For example, in case of re-hypothecation (same LGD),

\[ \Pi_{\text{DVAcoll}} = L_{\text{GD}I}(-(\varepsilon_{\tau} - C_{\tau^-}))^+, \quad \Pi_{\text{CVAcoll}} = L_{\text{GD}C}(\varepsilon_{\tau} - C_{\tau^-})^+. \]
Including credit, collateral, funding & multi-curve effects  Updating cash flows to include all effects

Adding costs of funding the trade through the treasury (and offsetting Repo market)

FVA can be sizeable too. JP Morgan:

Wall St Journal, Jan 14, 2014: ‘[…] So what is a funding valuation adjustment, (FVA) and why did it cost J.P. Morgan Chase $1.5 billion?’
Funding Costs of the trade accounts

As fourth contribution we consider the cost of funding (carry) for the trade accounts and we add the relevant cash flows ([91]).

\[ V_t := E_t \left[ \Pi + \gamma + 1_{\tau < T} D \theta + \varphi(t, T \land \tau; F, H) \right] \]

The last term is related to FVA.

- \( F_t \) is the cash account for the replication of the trade,
- \( H_t \) is the risky-asset account in the replication,
- \( \varphi \) is cost/benefit of carry for the cash \( F \) and hedging \( H \) accounts.
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- In classical Black Scholes for a call option,

\[ V_{t}^{\text{Call}} = \Delta_t S_t + \eta_t B_t =: H_t + F_t, \quad (\text{no } \tau, \ C = \gamma = \varphi = 0). \]
Including credit, collateral, funding & multi-curve effects  

Updating cash flows to include all effects

**Funding costs of the trade accounts**

\[
\varphi(t, \tau \wedge T) \approx - \sum_{j=1}^{m-1} 1_{\{t \leq t_j < \tau \wedge T\}} D(t, t_j) (F_{t_j} + H_{t_j}) \alpha_k \left( \tilde{f}_{t_j} - r_{t_j} \right)
\]

\[
+ \sum_{j=1}^{m-1} 1_{\{t \leq t_j < \tau \wedge T\}} D(t, t_j) H_{t_j} \alpha_k \left( \tilde{h}_{t_j} - r_{t_j} \right)
\]

\[\tilde{f} := f^+ 1_{\{F+H>0\}} + f^- 1_{\{F+H<0\}} \quad \tilde{h} := h^+ 1_{\{H>0\}} + h^- 1_{\{H<0\}}\]

- If \( H \) is perfectly collateralized with collateral re-hypothecation (\( H = 0 \) as we fund the hedge via its collateral)

\[
\varphi(t, \tau \wedge T) \approx \sum_{j=1}^{m-1} 1_{t \leq t_j < \tau \wedge T} D(t, t_j) \alpha_k \begin{bmatrix}
- (F_{t_j})^+ \left( f_{t_j}^+ - r_{t_j} \right) \\
E: Funding Cost FCA \\
\end{bmatrix} + \begin{bmatrix}
(F_{t_j})^+ \left( f_{t_j}^- - r_{t_j} \right) \\
E: Benefit FBA
\end{bmatrix}
\]

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Funding costs of the trade accounts

- If further treasury borrows/lends at risk free $\tilde{f} = r \Rightarrow \varphi = FVA = 0$. 

- Funding rates and bank policy $f + f -$ are policy driven. EG, $f^+ = r + s^I + \ell$, $f^- = r + s^I - (s^I$ is our bank spread, typically $s^I = \lambda I LD$), $\ell$ are liquidity bases driven by treasury policy and market (CDS-Bond basis).

- Credit risk drives all the adjustments. What explains the funding cost we pay to the treasury is the cost our bank faces in borrowing externally to service our trade. This cost is indexed at our bank credit risk. Margining is a machinery to transform credit risk in liquidity risk/costs, but credit risk has not disappeared.
Funding costs of the trade accounts

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Funding rates and bank policy

$f^+ \& f^- \text{ are policy driven. EG, } f^+ = r + s^I + \ell^+, f^- = r + s^I + \ell^- \text{ (} s^I \text{ is our bank spread, typically } s^I = \lambda^I L_{GD}^I, \ell \text{ are liquidity bases driven by treasury policy and market (CDS-Bond basis).}$

Credit risk drives all the adjustments

What explains the funding cost we pay to the treasury is the cost our bank faces in borrowing externally to service our trade. This cost is indexed at our bank credit risk.

Margining is a machinery to transform credit risk in liquidity risk/costs, but credit risk has not disappeared.
Putting everything together

\[
(*) \quad V_t = \mathbb{E}_t \left[\begin{array}{c}
\Pi(t, T \wedge \tau) + \gamma(t) + 1_{\{\tau < T\}} D(t, \tau) \theta_{\tau} + \varphi(t)
\end{array}\right]
\]

- Basic cash flows
- Collateral carry
- Closeout cashflows
- Funding carry

Nonlinear valuation & XVA
Putting everything together

\[(*) \quad V_t = \mathbb{E}_t \left[ \Pi(t, T \wedge \tau) + \gamma(t) + 1_{\{\tau<T\}} D(t, \tau) \theta_{\tau} + \varphi(t) \right] \]

Can we interpret:

\[\mathbb{E}_t \left[ \Pi + 1_{\{\tau<T\}} D(t, \tau) \theta \right] : \text{RiskFree Price + DVA - CVA?} \]
\[\mathbb{E}_t [ \gamma + \varphi ] : \text{Funding adjustment ColVA+FVA?} \]
Including credit, collateral, funding & multi-curve effects

The recursive non-decomposable nature of adjusted prices

Putting everything together

\[
(*) \quad V_t = \mathbb{E}_t \left[ \Pi(t, T \wedge \tau) + \gamma(t) + 1_{\{\tau < T\}} D(t, \tau) \theta_{\tau} + \varphi(t) \right]
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Can we interpret:

\[
\mathbb{E}_t \left[ \Pi + 1_{\{\tau < T\}} D(t, \tau) \theta \right] : \text{RiskFree Price + DVA - CVA?}
\]

\[
\mathbb{E}_t [\gamma + \varphi] : \text{Funding adjustment ColVA+FVA?}
\]

Not really. \(V_t = F_t + H_t + C_t\) (re–hypo) \(\Rightarrow \varphi\) term depends on future \(F = V - H - C\) to distinguish \(f^+\) & \(f^-\), & the closeout depends on future \(V\) too. All terms depend on \(V\) (all risks), no neat separation.
Including credit, collateral, funding & multi-curve effects

The recursive non-decomposable nature of adjusted prices

Putting everything together

\[
(*) \quad V_t = \mathbb{E}_t \left[ \Pi(t, T \wedge \tau) + \gamma(t) + 1_{\{\tau < T\}} D(t, \tau) \theta_{\tau} + \varphi(t) \right]
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Basic cash flows \hspace{1cm} \text{collateral carry} \hspace{1cm} \text{closeout cashflows} \hspace{1cm} \text{funding carry}

Can we interpret:

\[
\mathbb{E}_t \left[ \Pi + 1_{\{\tau < T\}} D(t, \tau) \theta \right] : \text{RiskFree Price + DVA - CVA?}
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Not really. \( V_t = F_t + H_t + C_t \) \hspace{1cm} \text{(re–hypo) } \Rightarrow \varphi \text{ term depends on future future.}

\( F = V - H - C \) to distinguish \( f^+ \) & \( f^- \), & the closeout depends on future \( V \) too. All terms depend on \( V \) (all risks), no neat separation.

Nonlinear Equation! \hspace{1cm} (FBSDE, SL-PDE)

\[
V_t = \mathbb{E}_t \left[ \Pi + \gamma(t, V) + 1_{\{\tau < T\}} D(t, \tau) \theta_{\tau}(V) + \varphi(t, V) \right]
\]
Including credit, collateral, funding & multi-curve effects

Semi-linear PDEs & BSDEs: Existence, Uniqueness, Invariance

This last Eq. can be written as a FBSDE driven by a Brownian martingale by switching to pre-default $\mathcal{F}$ expectations

- **Key-Lemma**
  \[
  \mathbb{E}\left[1_{\{t<\tau\leq s\}}X \mid \mathcal{G}_t\right] = 1_{\{\tau>t\}} \frac{\mathbb{E}\left[1_{\{t<\tau\leq s\}}X \mid \mathcal{F}_t\right]}{\mathbb{E}\left[1_{\{\tau>t\}} \mid \mathcal{F}_t\right]}
  \]

- For each process $x_t$ consider its $\mathcal{F}$-predictable version
  \[
  \tilde{x}_t 1_{\{\tau>t\}} = x_t 1_{\{\tau>t\}}
  \]

- $\mathcal{F}$-conditionally independent default times generated by Cox processes with intensities $\lambda^I$ and $\lambda^C$. $\lambda = \lambda^I + \lambda^C$
This last Eq. can be written as a FBSDE driven by a Brownian martingale by switching to pre-default $\mathcal{F}$ expectations

- **Key-Lemma** \[ \mathbb{E}\left[ 1_{\{t<\tau\leq s\}} X \mid \mathcal{G}_t \right] = 1_{\{\tau>t\}} \frac{\mathbb{E}\left[ 1_{\{t<\tau\leq s\}} X \mid \mathcal{F}_t \right]}{\mathbb{E}\left[ 1_{\{\tau>t\}} \mid \mathcal{F}_t \right]} \]

- For each process $x_t$ consider its $\mathcal{F}$-predictable version
  \[ \tilde{x}_t 1_{\{\tau>t\}} = x_t 1_{\{\tau>t\}} \]

- $\mathcal{F}$-conditionally independent default times generated by Cox processes with intensities $\lambda^I$ and $\lambda^C$. $\lambda = \lambda^I + \lambda^C$

The above then becomes the following BSDE if $\mathcal{F} = \sigma(\mathcal{W})$:

\[
\begin{aligned}
    d\tilde{V}_u + & \left[ \pi_u - (f_u + \lambda_u)\tilde{V}_u + \tilde{\theta}_u + (f_u - c_u)C_u - (r_u - h_u)\tilde{H}_u \right] du = Z_u dW_u \\
    \tilde{\theta}_u = & \varepsilon_u \lambda_u - LGD_C(\varepsilon_u - C_u)^+ \lambda_u^C + LGD_I(-(\varepsilon_u - C_u))^+ \lambda_u^I.
\end{aligned}
\]

Under a Markovian/generalized Delta hedging framework one can obtain a PDE from the BSDE. Depending on assumptions this may have viscosity or classical solutions.
Example of PDE generalizing Black Scholes

See e.g. B. & al [30] and Capponi et al. [11] for \( \exists \) & uniqueness of solutions. B. et al [30] emphasize also invariance wrt \( r \). B. & al [22] emphasize the Black Scholes case in detail. To approach B&S assume \( C_t = \alpha_t V_t \), \( \lambda \)'s are deterministic, \( h = f \), \( H_t = S_t \partial_S V_t \). Then

\[
\partial_t V - f_+ (V - S_t \partial_S V_t - \alpha V)^+ + f_- (-V + S_t \partial_S V_t + \alpha V)^+ - \lambda_t V + \\
\frac{1}{2} \sigma^2 S^2 \partial_S^2 V - c_+ \alpha_t (V_t)^+ + c_- \alpha_t (-V_t)^+ + \pi_t + \lambda_t \theta_t (V_t) = 0
\]

- if \( f_+ = f_- = r \) (symmetric risk free borrowing and lending),
- \( \alpha = 0 \) (no collateral), \( \lambda = 0 \) (no credit risk),

then we get back the Black Scholes LINEAR (parabolic) PDE.

\[
\partial_t V + r S_t \partial_S V_t + \frac{1}{2} \sigma^2 S^2 \partial_S^2 V - r V + \pi = 0.
\]
Kapital Valuation Adjustment. Replication?

Strengthening of capital requirements induced banks to consider charging a so-called capital valuation adjustment (KVA) to clients.

Initial attempt [69] to embed capital costs in “replication” framework as above, adding “capital account”. Dubious (already for FVA...).

Alternative approach in B. et al [31] is relating costs of capital to ex-ante risk-constrained target performance of bank. E.g. ex-ante RAROC = \text{Expected P&L} / \text{VaR} (Sironi et al [101]) could play a role.

When starting new trade with client, this generates new risk \text{VaR}.

This reduces expected target performance. The improved \text{P&L} from the new trade may not be enough to compensate the new risk.

To keep target as before bank needs charge a profit margin.

(CVA is subject to capital: KVA on CVA? Additive? Nonlinear again?)
Indifference approach to cost of capital constraints I

One period model. Bank is endowed with some cash $C_0$, and there is a liquid arbitrage-free market, with $d$ securities with vector prices $S_0$ (deterministic) at $t = 0$ and $S_1$ (random) at $t = 1$.

A strategy on the liquid market is $\theta \in \mathbb{R}^d$.

Bank borrows & lends money at spread $\lambda$ over the risk-free rate $r$. This is for computational convenience, easily generalized to different $\lambda$’s.

Aside from liquid market, bank consider a OTC trade with payoff $qY$ ($q$ amount of product, $Y$ product payoff) with a counterparty.

Counterparty interested in buying from the bank $q$ units of payoff $Y$ and they ask the bank for a quote on this product.

Problem: what is the price $P(q)$ that the bank is willing to make to the counterparty for the contract with pay-off $Y$. 
Indifference approach to cost of capital constraints II

To proceed more in detail, for our chosen strategy $\theta$ we have:

\[ \text{Assets} = (\theta^T S_1)^+ + (qY)^+ + (C_0 - \theta^T S_0 + P(q))^+(1 + r + \lambda^B); \]

\[ \text{Liabilities} = (\theta^T S_1)^- + (qY)^- + (C_0 - \theta^T S_0 + P(q))^- (1 + r + \lambda^B). \]

For convenience define bank equity at $t = 1$ as assets – liabilities:

\[ X(\theta, q) = qY + \theta^T (S_1 - S_0(1 + r + \lambda)) + (C_0 + P(q))(1 + r + \lambda^B). \]

Assume VaR is proportional to the standard deviation:

\[
RAROC(\theta, q) = \frac{\mathbb{E}[X(\theta, q)] - C_0}{m \sqrt{\text{Var}[X(\theta, q)]}}.
\] (1)

Find $P(q)$ s.t. $RAROC(\theta, 0) = RAROC(\theta', q)$, where $\theta'$ is strategy that the bank chooses in the case when it also trades the new product.
Indifference approach to cost of capital constraints III

Bank chooses $\theta$ to maximise value. Admissible set $\Theta(q) \subseteq \mathbb{R}^d$

\[ \bar{v}(q) = \sup_{\theta \in \Theta(q)} \mathbb{E} [X(\theta, q)] . \] (2)

\[ \Theta(q) = \left\{ \theta \in \mathbb{R}^2 \mid C_0 = \nu \sqrt{\theta^T A \theta + 2q \theta^T b + \sigma_Y^2 q^2} \right\} , \] (3)

where $\sigma_Y^2$ is the variance of the payoff $Y$, and

\[ \text{Cov} \left[ \begin{pmatrix} S_1 \\ Y \end{pmatrix} \right] = \begin{pmatrix} A & b \\ b^T & \sigma_Y^2 \end{pmatrix} . \]

We are computing the capital constraint as an ES or V@R limit approximated with a multiple $\nu$ of the standard deviation of $X(\theta, q)$. 
We are supposing that the capital constraint is binding, or that is optimal for the bank to have the minimum amount of capital required.

Note: here endowment $C_0$ plays also role of capital of the bank: we are optimising the whole portfolio of the bank over one period.

\[
\text{We solve } \tilde{\nu}(0) = \tilde{\nu}(q) \text{ (indifference to new position)}
\]

The treatment of the maximisation problem (2)
\[
\tilde{\nu}(q) = \sup_{\theta \in \Theta(q)} \mathbb{E} [X(\theta, q)]
\]
is classic (see for example [57, 66]).
Indifference approach to cost of capital constraints V

Main result for whole-bank view. Set $\mu = \mathbb{E}[(S_1 - S_0(1 + r + \lambda))]$. The indifference Equation has a solution that is given by

$$P(q) \simeq D^\lambda \left(-q\mathbb{E}[Y] + \left(\frac{(\sigma_Y^2 - b^T A^{-1} b)q^2 \nu}{C_0}\right) \frac{\sqrt{\mu^T A^{-1} \mu}}{2} + qb^T A^{-1} \mu\right)$$

The price that would make the bank break even is sum of 3 terms:

1. minus the expectation of the payoff under $\mathbb{P}$;
2. a correction (KVA?) due to the presence of the capital constraint; the coefficient $\sqrt{\mu^T A^{-1} \mu} = \nu \text{RAROC} - (r + \lambda)$ (next slide).
3. the expected excess return of the hedging portfolio, i.e. the difference between the expected value of the hedging portfolio at time 1 and its price at time 0 accrued at $1 + r + \lambda$. 

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Indifference approach to cost of capital constraints VI

Our expected P&L, associated with the best strategy, is:

\[ \tilde{v}(0) - C_0 = \left( \frac{\mu^T A^{-1}}{2\chi(0)} \right) \mu + C_0(r + \lambda) = \sqrt{\mu A^{-1} \mu} \frac{C_0}{\nu} + C_0(r + \lambda) \]

To simplify our formulae we choose the same ES metric we used as capital constraint as denominator for our RAROC. Hence we have that:

\[ RAROC(\tilde{\theta}, 0) = \frac{\sqrt{\mu A^{-1} \mu} \frac{C_0}{\nu} + C_0(r + \lambda)}{C_0} = \frac{\sqrt{\mu A^{-1} \mu}}{\nu} + r + \lambda =: h. \]
Indifference approach: Shareholders perspective I

While in the previous part we took the whole bank perspective, in [31] we analyse also how capital constraints play a role in the shareholder’s valuation of a deal.

Shareholder’s objective function is different from the whole-bank one, since in case of default (negative equity at time 1 in our model) they have limited liability. This means that the best strategy for a shareholder solves the following optimisation problem:

\[ v(q) = \sup_{\theta \in \Theta(q)} E \left[ (X(\theta, q))^+ \right]. \]

Note the positive part, that was missing in the whole bank case.
Indifference approach: Shareholders perspective II

The analysis of $v(q) = v(0)$ in this case is possible only approximately.

$$v(q) = v(0) + q \frac{dv}{dq} \bigg|_{q=0} + q^2 \frac{1}{2} \frac{d^2v}{dq^2} \bigg|_{q=0} + o(q^2),$$

and, inspired from our linearization based result for whole bank, we look for price $P$ as a second degree polynomial in $q$ such that

$$q \frac{dv}{dq} \bigg|_{q=0} + q^2 \frac{1}{2} \frac{d^2v}{dq^2} \bigg|_{q=0} = 0.$$

This is a 2nd order extension of what in the literature is usually called *marginal price*. We find a closed form solution in B. et al [31].

Finally, in B. et al [31] we show that if we take median instead of $\mathbb{E}$ in indifference optimization, then the whole bank and shareholder cases have the same solution we showed earlier.
Nonlinearities

Should we embrace nonlinearities or keep them at arm’s length?

**LINEAR VALUATION**

\[ V = V_{\text{basic}} - \text{CVA} - \text{FCA + FBA} - \text{KVA} \]

- Resp. for standard trades
- Resp. for credit valuation
- Resp. for funding valuation
- Resp. for Capital Costs
Nonlinearity and responsibility

NONLINEAR VALUATION

V non separable: Basic flows, credit, funding, capital

Every trader responsible for every risk. Ok with enlightened people or with Bees... ZZZZZZZZ but with us normal humans? Management problem
Nonlinearity: linearize?

Looks like managers may wish to linearize!
EG, symmetrize borrowing & lending rates, risk free closeout at default.
Nonlinearity: linearize?

Looks like managers may wish to linearize!
EG, symmetrize borrowing & lending rates, risk free closeout at default.

Nonlinearity Valuation Adjustment (NVA) (B. et al (2014)[32])

NVA analyzes the double counting involved in forcing linearization. Our numerical examples for simple call options show that NVA can easily reach 2 or 3% of deal value even in standard settings without WWR.
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Looks like managers may wish to linearize!
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Multiple interest rate curves

Full theory may account also for multiple discount curves (see [90]).
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Multiple interest rate curves

Full theory may account also for multiple discount curves (see [90]).

Initial Margins (CCPs, SCSA)

Add the initial margin account flows, liquidation delays & customize to relevant (CCP/SCSA) initial margin. IMVA [45].
Price vs Value

Cecil Graham: “What is a cynic?”
Lord Darlington: “A man who knows the **price** of everything, and the **value** of nothing.”

Oscar Wilde, Lady Windermere’s Fan
Price vs Value

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Oscar Wilde, Lady Windermere’s Fan

Price of Value? Charging clients?

Adjusted ”price”? OK for cost/ profitability analysis or internal fund transfers, but can we charge to client straight away?

How can client check our “price” is fair if she has no access to our funding policy, target profit & parameters? More Value than Price.

What if the client does not approve our profit target or funding policy?

Go to other bank? Banks herd behaviour/self-fulfilling prophecy?
Conclusions

Will new valuation adjustments appear, widening gap *price-value*, confusing the picture further, increasing risk of double counting?
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Coming soon: **EVA** (Electricity-bill VA’s)

**What of the Quant profession?**
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What of the Quant profession?

Yesterday ← The Valuation Quant → Today

PhD in pure mathematics, physics... little/no knowledge of:

how bank works, profit targets, regulation, accounting standards, trading costs, finance & economics, Fintech & industrialization.

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Thank you for your attention! Questions?
Disclaimer

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Capital valuation adjustment and funding valuation adjustment.  
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Funding value adjustments.  
2016.

[4] Basel Committee on Banking Supervision
Available at http://www.bis.org.


References III


References IV


References VIII


References XIII


References XVI


Simplified mean-variance portfolio optimisation.


References XX


References XXII


Bonus material

The following material did not fit the talk due to time limits.

I include it here for potential questions and follow up.
A Trader’s explanation of the funding cash flows

1. **Time** $t$: I wish to buy a call option with maturity $T$ whose current price is $V_t = V(t, S_t)$. I need $V_t$ cash to do that. So I borrow $V_t$ cash from my bank treasury and buy the call.
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3. Now I wish to hedge the call option I bought. To do this, I plan to repo-borrow $\Delta_t = \partial_S V_t$ stock on the repo-market.
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4. To do this, I borrow $H_t = \Delta_t S_t$ cash at time $t$ from the treasury.
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5. I repo-borrow an amount $\Delta_t$ of stock, posting cash $H_t$ guarantee.
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5. I repo-borrow an amount \( \Delta_t \) of stock, posting cash \( H_t \) guarantee.

6. I sell the stock I just obtained from the repo to the market, getting back the price \( H_t \) in cash.
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7. I give \( H_t \) back to treasury.
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7. I give $H_t$ back to treasury.

8. Outstanding: I hold the Call; My debt to the treasury is $V_t - C_t$; I am Repo borrowing $\Delta_t$ stock.
A Trader’s explanation of the funding cash flows

**Time** $t + dt$: I need to close the repo. To do that I need to give back $\Delta_t$ stock. I need to buy this stock from the market. To do that I need $\Delta_t S_{t+dt}$ cash.
A Trader’s explanation of the funding cash flows

9 **Time** \( t + dt \): I need to close the repo. To do that I need to give back \( \Delta_t \) stock. I need to buy this stock from the market. To do that I need \( \Delta_t S_{t+dt} \) cash.

10 I thus borrow \( \Delta_t S_{t+dt} \) cash from the bank treasury.
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11 I buy $\Delta_t$ stock and I give it back to close the repo and I get back the cash $H_t$ deposited at time $t$ plus interest $h_t H_t$. 

Notice that this $-\Delta_t dS_t$ is the right amount I needed to hedge $V$ in a classic delta hedging setting.

I close the derivative position, the call option, and get $V_t + dt$ cash.
A Trader’s explanation of the funding cash flows

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12 I give back to the treasury the cash $H_t$ I just obtained, so that the net value of the repo operation has been

$$H_t(1 + h_t dt) - \Delta_t S_{t+dt} = -\Delta_t dS_t + h_t H_t \ dt$$

Notice that this $-\Delta_t dS_t$ is the right amount I needed to hedge $V$ in a classic delta hedging setting.
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13 I close the derivative position, the call option, and get $V_{t+dt}$ cash.
I have to pay back the collateral plus interest, so I ask the treasury the amount $C_t(1 + c_t \, dt)$ that I give back to the counterparty.
A Trader’s explanation of the funding cash flows

14 I have to pay back the collateral plus interest, so I ask the treasury the amount \( C_t(1 + c_t \, dt) \) that I give back to the counterparty.

15 My outstanding debt plus interest (at rate \( f \)) to the treasury is 
\[
V_t - C_t + C_t(1 + c_t \, dt) + (V_t - C_t)f_t \, dt = V_t(1 + f_t \, dt) + C_t(c_t - f_t \, dt).
\]
I then give to the treasury the cash \( V_{t+dt} \) I just obtained, the net effect being

\[
V_{t+dt} - V_t(1 + f_t \, dt) - C_t(c_t - f_t) \, dt = dV_t - f_t \, V_t \, dt - C_t(c_t - f_t) \, dt
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A Trader’s explanation of the funding cash flows

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16 I now have that the total amount of flows is:

$$-\Delta_t \, dS_t + h_t H_t \, dt + dV_t - f_t V_t \, dt - C_t(c_t - f_t) \, dt$$
Now I present–value the above flows in $t$ in a risk neutral setting.

$$
\mathbb{E}_t[-\Delta_t dS_t + h_t H_t \, dt + dV_t - f_t V_t \, dt - C_t(c_t - f_t) \, dt] = \\
= -\Delta_t (r_t - h_t) S_t \, dt + (r_t - f_t) V_t \, dt - C_t(c_t - f_t) \, dt - d\varphi(t) \\
= -H_t(r_t - h_t) \, dt + (r_t - f_t)(H_t + F_t + C_t) \, dt - C_t(c_t - f_t) \, dt - d\varphi(t) \\
= (h_t - f_t)H_t \, dt + (r_t - f_t)F_t \, dt + (r_t - c_t)C_t \, dt - d\varphi(t)
$$

This derivation holds assuming that $\mathbb{E}_t[dS_t] = r_t S_t \, dt$ and $\mathbb{E}_t[dV_t] = r_t V_t \, dt - d\varphi(t)$, where $d\varphi$ is a dividend of $V$ in $[t, t + dt)$ expressing the funding costs. Setting the above expression to zero we obtain

$$
d\varphi(t) = (h_t - f_t)H_t \, dt + (r_t - f_t)F_t \, dt + (r_t - c_t)C_t \, dt
$$

which coincides with the definition given earlier.
Funding inclusive valuation equations I

Under filtration $\mathcal{G}$ we have under re-hypotecation assumption:

$$V_t = \mathbb{E} \left[ \int_t^{\tilde{\tau}} D(t, u) \pi_u du + D(t, \tau) 1_{\{t < \tau < T\}} \theta_\tau \mid \mathcal{G}_t \right]$$

$$+ \mathbb{E} \left[ \int_t^{\tilde{\tau}} D(t, u)((f_u - c_u) C_{(u)} + (r_u - f_u) V_{(u)} - (r_u - h_u) H_{(u)}) du \mid \mathcal{G}_t \right]$$

We switch to $\mathcal{F}$ expectations

- **Key-Lemma** $\mathbb{E} \left[ 1_{\{t < \tau \leq s\}} X \mid \mathcal{G}_t \right] = 1_{\{\tau > t\}} \frac{\mathbb{E} \left[ 1_{\{t < \tau \leq s\}} X \mid \mathcal{F}_t \right]}{\mathbb{E} \left[ 1_{\{\tau > t\}} \mid \mathcal{F}_t \right]}$

- For each process $x_t$ consider its $\mathcal{F}$-predictable version

$$\tilde{x}_t 1_{\{\tau > t\}} = x_t 1_{\{\tau > t\}}$$

- $\mathcal{F}$-conditionally independent default times generated by Cox processes with deterministic intensities $\lambda^I$ and $\lambda^C$. $\lambda = \lambda^I + \lambda^C$
Funding inclusive valuation equations II

Under $\mathcal{F}$ we obtain:

$$
\tilde{V}_t = \mathbb{E}\left[ \int_t^T D(t, u, r + \lambda)(\pi_u + \tilde{\theta}_u + (f_u - c_u)C_u)du \mid \mathcal{F}_t \right].
$$

\[ + \mathbb{E}\left[ \int_t^T D(t, u, r + \lambda)((r_u - f_u)\tilde{V}_u - (r_u - h_u)\tilde{H}_u)du \mid \mathcal{F}_t \right] \]

That is in fact equivalent to the following BSDE if $\mathcal{F} = \sigma(\mathcal{W})$:

$$
d\tilde{V}_u + \left[ \pi_u - (f_u + \lambda_u)\tilde{V}_u + \tilde{\theta}_u + (f_u - c_u)C_u - (r_u - h_u)\tilde{H}_u \right] du = Z_udW_u
$$

Where

$$
\tilde{\theta}_u = \varepsilon_u\lambda_u - LGD_C(\varepsilon_u - C_u)^+\lambda_u^C + LGD_l(-(\varepsilon_u - C_u))^+\lambda_u^l.
$$
Funding inclusive valuation equations III

Markovian framework:

- Black-Scholes type dynamics for underlying $S_u$
- $\pi_u$ deterministic function $\pi(u, S_u)$ of $u$ and $S_u$, Lipschitz continuous in $S_u$
- the rates $r, f^\pm, c^\pm, \lambda^l, \lambda^C, h^\pm$ are deterministic bounded functions of time
- collateral $C_u = \alpha_u \tilde{V}_u$, where $0 \leq \alpha_u \leq 1$
- the close-out value $\varepsilon_t$ is equal to $\tilde{V}_t$ (adds a source of non-linearity)

Delta-Hedging

We choose $\tilde{H}_t = S_t \frac{Z_t}{\sigma(t, S_t)}$; if we suppose $\tilde{V}_t = V(t, S_t)$ with $V(\cdot, \cdot) \in C^{1,2}$ applying Ito’s formula we have $\sigma(t, S_t) \partial_S V(t, S_t) = Z_t$. 
Funding inclusive valuation equations IV

If we set \( \eta_t = f_t(\alpha_t - 1) - \lambda_t - c_t \alpha_t \) we have the decoupled FBSDE:

\[
\begin{align*}
    dS_t &= r_t S_t dt + \sigma(t, S_t) dW_t, \\
    d\tilde{V}_t &= - \left[ \pi_t + \tilde{\theta}_t + \eta_t \tilde{V}_t - (r_t - h_t) \frac{S_t Z_t}{\sigma(t, S_t)} \right] dt + Z_t dW_t, \\
    \tilde{V}_T &= 0
\end{align*}
\]

Under suitable conditions: \( V_t = u(t, S_t), \quad Z_t = \sigma(t, S_t) \frac{\partial_s u(t, S_t)}{\partial_s u(t, S_t)} \)

where \( u(t, s) \) satisfies the associated PDE

\[
\begin{cases}
    \partial_t u(t, s) + \frac{1}{2} \sigma(t, s)^2 \partial_s^2 u(t, s) + h_t s \partial_s u(t, s) + B'(t, s, u(t, s)) = 0 \\
    u(T, s) = 0 \quad \text{with} \quad B' = B + r_t s \partial_s u(t, s)
\end{cases}
\]

**Invariance Theorem:** The PDE does NOT depend on the risk-free rate \( r_t \) (blue terms cancel). Hence our price does not depend on the risk-free rate. No need for proxying \( r \) with OIS or others.
Existence and Uniqueness Results

- Under Lipschitz conditions in $v, z$ on $B$ and $\sigma$ we can apply the seminal result of Pardoux and Peng (1992) and obtain an existence and uniqueness result for our FBSDE and the existence and uniqueness of a viscosity solution to our PDE.

- We can obtain existence and uniqueness of classical solution for our PDE by requiring more regularity just on the drift of the forward equation, e.g. using Zhang (2001). To do so we need to find an equivalent FBSDE moving the hedging term into the forward equation. (Details in B., Francischello and Pallavicini (2015) [30] \url{http://arxiv.org/abs/1506.00686})

Classical solutions $\Rightarrow$ Ito’s formula $\Rightarrow Z_t$ is the Delta Hedging process
Using Feynman Kac formally (but with the confidence coming from the \( \exists ! \) of the solution) we can also write the valuation formula as

\[
\tilde{V}_t = \int_t^T \mathbb{E}^h \{ D(t, u; f)[\pi_u + (\tilde{\theta}_u - \lambda_u \tilde{V}_u) + (f_u - c_u)C_u] | \mathcal{F}_t \} du
\]

- This equation depends only on market rates, no theoretical \( r_t \)
- \( Q^h \) is the probability measure where the drift of the risky assets is \( h \), the repo rate, that in turn depends on \( H \) and hence on \( V \) itself. Nonlinearity. Deal dependent measure.
- We discount at funding. Note that \( f \) depends on \( V \), non-linearity. Deal dependent discount curve.
- \( \theta_u \) are trading CVA and DVA after collateralization
- \( (f_u - c_u)C_u \) is the cost of funding collateral with the treasury
- NO Explicit funding term for the replica as this has been absorbed in the discount curve and in the collateral cost
Treasury CVA & DVA I

Include default risk of funder & funded $\psi$, leading to $\text{CVA}_F$ & $\text{DVA}_F$.

$$V_t = \mathbb{E}_t \left[ \Pi(t, T \wedge \tau) + \gamma(t) + 1_{\{\tau < T\}} D(t, \tau) \theta_{\tau} + \varphi(t) + \psi(t, \tau_F, \tau) \right]$$
The benefit of lending back to the treasury, two different models:

1. **External funding benefit (EFB) policy:** whenever trading desk lends back to the treasury, the latter lends to F for interest \( f^- = r + s^F + \ell^- \). Hence

\[
\psi(t) = D(t, \tau)1_{\{\tau_{\tau_I} < T\}} \text{LGD}_{\tau_I}(F_{\tau_I})^+ - D(t, \tau_F)1_{\{\tau_{\tau_F} < T\}} \text{LGD}_F(-F_{\tau_F})^+
\]

and

\[
\bar{V}_{\text{EFB}} = V^0 - \text{CVA} + \text{DVA} + \text{LVA} - FCA - \underbrace{\text{DVA}_F}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{\text{CVA}_F + FBA_\ell} + \text{DVA}_F - \text{CVA}_F
\]
Reduced borrowing benefit (RBB) policy: whenever trading desk lends back to the treasury, the latter reduces the desk loan outstanding and the desk saves at interest \( f^- = r + s^I + \ell^- \). Hence

\[
\psi(t) = D(t, \tau) \mathbb{1}_{\{\tau = \tau_I < T\}} LGD_I(F_{\tau_I})^+ - D(t, \tau_F) \mathbb{1}_{\{\tau = \tau_F < T\}} LGD_F(-F_{\tau_F})^+
\]

and

\[
\bar{V}_{RBB} = V^0 - CVA + DVA + LVA - FCA - DVA - FCA_{\ell} + FBA + FBA_{\ell} + DVA_F
\]
Treasury CVA & DVA

Include default risk of funder and funded $\psi$, leading to $\text{CVA}_F$ & $\text{DVA}_F$.

$$V_t = \mathbb{E}_t \left[ \Pi(t, T \land \tau) + \gamma(t) + 1_{\{\tau < T\}} D(t, \tau) \theta_{\tau} + \varphi(t) + \psi(t, \tau_F, \tau) \right]$$
Treasury CVA & DVA

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FVA = $-\text{FCA} + \text{FBA}$ from $f^+$ & $f^-$ largely offset by $\mathbb{E}\psi$ after immersion, approx & linearization.
Treasury CVA & DVA

Include default risk of funder and funded $\psi$, leading to $\text{CVA}_F$ & $\text{DVA}_F$.

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FVA = $-\text{FCA} + \text{FBA}$ from $f^+$ & $f^-$ largely offset by $\mathbb{E}_t \psi$ after immersion, approx & linearization.

Assume $H = 0$ (perfectly collateralized hedge with re-hypothecation), once $f^+$ & $f^-$ are decided by policy, under *immersion*. 
Treasury CVA & DVA

Include default risk of funder and funded $\psi$, leading to $\text{CVA}_F$ & $\text{DVA}_F$.

$$V_t = \mathbb{E}_t \left[ \Pi(t, T \wedge \tau) + \gamma(t) + 1_{\{\tau < T\}} D(t, \tau) \theta_\tau + \varphi(t) + \psi(t, \tau_F, \tau) \right]$$

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Assume $H = 0$ (perfectly collateralized hedge with re–hypothecation), once $f^+$ & $f^-$ are decided by policy, under immersion

- Underlying $\Pi(t, T)$ is not credit sensitive, technically $\mathcal{F}_t$-measurable; $\mathcal{F}$ pre-default filtration, $\mathcal{G}$ full filtration.
Treasury CVA & DVA

Include default risk of funder and funded $\psi$, leading to $\text{CVA}_F$ & $\text{DVA}_F$.

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Assume $H = 0$ (perfectly collateralized hedge with re-hypothecation), once $f^+$ & $f^-$ are decided by policy, under immersion

- Underlying $\Pi(t, T)$ is not credit sensitive, technically $\mathcal{F}_t$-measurable; $\mathcal{F}$ pre-default filtration, $\mathcal{G}$ full filtration.

- $\tau_I$ and $\tau_C$ and $\tau_F$ are $\mathcal{F}$ conditionally independent (credit spreads can be correlated, jumps to default are independent);

we obtain a practical decomposition of price into
Extra Material

What if FVA=0?

\[ V = \text{RiskFreePrice} - \text{CVA} + \text{DVA} + \text{LVA} - \text{FCA} + \text{FBA} - \text{CVA}_F + \text{DVA}_F \]

\[ \text{RiskFreePr} = \int_t^T \mathbb{E}\left\{ \pi_u | \mathcal{F}_t \right\} du; \quad \text{LVA} = \int_t^T \mathbb{E}\left\{ D(t, u; r+\lambda)(r_u-\tilde{c}_u)C_u | \mathcal{F}_t \right\} du \]
$V = \text{RiskFreePrice} - \text{CVA} + \text{DVA} + \text{LVA} - \text{FCA} + \text{FBA} - \text{CVA}_F + \text{DVA}_F$

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\[ -\text{CVA} = \int_t^T \mathbb{E}\left\{ D(t, u; r + \lambda) \left[ -L_{GD}\lambda C(u)(V_u - C_u)^+ \right] | \mathcal{F}_t \right\} du \]
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\[ \text{DVA} = \int_t^T \mathbb{E}\{ D(t, u; r + \lambda) \left[ L_{GD} I \lambda_I(u)(-(V_u - C_u)) \right] | \mathcal{F}_t \} \, du \]
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\]

\[
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\[
DVA = \int_t^T \mathbb{E}\left\{ D(t, u; r + \lambda)\left[ L_{GD} I\lambda_I(u)(- (V_u - C_{u-}))^+ \right] \mid \mathcal{F}_t \right\} du
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-CVA_F = \int_t^T \mathbb{E}\left\{ D(t, u; r + \lambda)\left[ L_{GD} F\lambda_F(u)(- (V_u - C_u))^+ \right] \mid \mathcal{F}_t \right\} du
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What if FVA=0?

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\[ DVA = \int_t^T \mathbb{E}\left\{ D(t, u; r + \lambda)\left[ L_{GD}I\lambda_I(u)(- (V_u - C_u^-))^+ \right] | \mathcal{F}_t \right\} du \]

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Treasury CVA & DVA

To further specify the split, we need to assign $f^+$ (borrow) & $f^-$ (lend). There are two possible simple treasury models to assign $f$. 
Treasury CVA & DVA

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There are two possible simple treasury models to assign \( f \).

\[
f^+ = L_{GD_I} \lambda_I + \ell^+ =: s_I + \ell^+
\]
\[
f^- = L_{GD_F} \lambda_F + \ell^- =: s_F + \ell^-
\]

\[
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\]
\[
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\]
Treasury CVA & DVA

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EFB: $s_F + \ell^-$; RBB: $s_I + \ell^-$
Treasury CVA & DVA

\[-CVA = \int_t^T \mathbb{E}\left\{ D(t, u; r + \lambda) \left[ -s_C(u)(V_u - C_u^-)^+ \right] | \mathcal{F}_t \right\} \, du \]

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\[FBA = \int_t^T \mathbb{E}\left\{ D(t, u; r + \lambda) \left[ (f_u^- - r_u \underbrace{\quad}_{\text{EFB: } s_F + \ell^-; \text{ RBB: } s_I + \ell^-}) (- (V_u - C_u))^+ \right] | \mathcal{F}_t \right\} \, du \]

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Treasury CVA & DVA

The benefit of lending back to the treasury, two different models:

1. External funding benefit (EFB) policy: when desk lends back to treasury, treasury lends to F for interest $f^- = r + s^F + \ell^-$. Hence

$$V_{\text{EFB}} = V^0 - CVA + DVA + ColVA - FCA + FBA + DVA_F - CVA_F$$

- $-DVA_F - FCA_{\ell}$
- $CVA_F + FBA_{\ell}$
The benefit of lending back to the treasury, two different models:

1. **External funding benefit (EFB) policy:** when desk lends back to treasury, treasury lends to F for interest $f^- = r + s^F + \ell^-$. Hence

$$V_{EFB} = V^0 - CVA + DVA + ColVA - FCA + FBA + DVA_F - CVA_F$$

2. **Reduced borrowing benefit (RBB) policy:** whenever trading desk lends back to the treasury, the latter reduces the desk loan outstanding and the desk saves at interest $f^- = r + s^l + \ell^-$. Hence

$$V_{RBB} = V^0 - CVA + DVA + ColVA - FCA + FBA + DVA_F$$
A lot of discussion on FVA. But what if I told you...

<table>
<thead>
<tr>
<th>Hull White argued FVA = 0.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modigliani Miller? (MM)</td>
</tr>
<tr>
<td>Mkt prices follow rnd walks,</td>
</tr>
<tr>
<td>No taxes, No costs for</td>
</tr>
<tr>
<td>bankruptcy or agency,</td>
</tr>
<tr>
<td>No asymmetric information,</td>
</tr>
<tr>
<td>&amp; market is efficient</td>
</tr>
<tr>
<td>then value of firm does not depend on how firm is financed.</td>
</tr>
</tbody>
</table>

Without MM or corporate finance:

\[
V_{\text{EFB}} = V_0 - CVA + DVA + \text{ColVA} - FCA \downarrow \uparrow - DVA F - FCA \ell + FBA \downarrow \uparrow CVA F + FBA \ell + DVA F - CVA F
\]

If bases $\ell = 0$, & if $r = \tilde{c}$, & if...

\[
V_{\text{EFB}} = V_0 - CVA + DVA (\text{no funding})
\]

Too many if’s? Even then, internal fund transfers happening.
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Without MM or corporate finance:

\[ V_{EFB} = V_0 - CVA + DVA + ColVA - FCA \]

\[ - DVA_F - FCA_{\ell} + FBA \]

\[ + CVA_F + FBA_{\ell} + DVA_F - CVA_F \]

If bases \( \ell = 0 \), & if \( r = \tilde{c} \), & if...

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Too many if’s? Even then, internal fund transfers happening.
NVA: numerical example I

Equity call option (long or short), \( r = 0.01, \sigma = 0.25, S_0 = 100, K = 80, T = 3y, \) \( V_0 = 28.9 \) (no credit risk or funding/collateral costs). Precise credit curves are given in the paper.

\[
NVA = V_0(\text{nonlinear}) - V_0(\text{linearized})
\]

Table: NVA with default risk and collateralization

<table>
<thead>
<tr>
<th>Funding Rates bps</th>
<th>Default risk, low(^a)</th>
<th>Default risk, high(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f^+ )</td>
<td>( f^- )</td>
<td>( \hat{f} )</td>
</tr>
<tr>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>300 100 200</td>
<td>-3.27 (11.9%)</td>
<td>-3.60 (10.5%)</td>
</tr>
<tr>
<td>100 300 200</td>
<td>3.63 (10.6%)</td>
<td>3.25 (11.8%)</td>
</tr>
</tbody>
</table>

The percentage of the total call price corresponding to NVA is reported in parentheses.

\(^a\) Based on the joint default distribution \( D_{\text{low}} \) with low dependence.

\(^b\) Based on the joint default distribution \( D_{\text{high}} \) with high dependence.
**NVA: numerical example II**

**Table: NVA with default risk, collateralization and rehypothecation**

<table>
<thead>
<tr>
<th>Funding Rates bps</th>
<th>Default risk, low&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Default risk, high&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Long</td>
<td>Short</td>
</tr>
<tr>
<td>$f^+$</td>
<td>-4.02 (14.7%)</td>
<td>-4.45 (12.4%)</td>
</tr>
<tr>
<td>$f^-$</td>
<td>4.50 (12.5%)</td>
<td>4.03 (14.7%)</td>
</tr>
<tr>
<td>$\hat{f}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
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NVA for long call as a function of $f^+ - f^-$, with $f^- = 1\%$, $f^+$ increasing over 1% and $\hat{f}$ increasing accordingly. NVA expressed as an additive price component on a notional of 100, risk free option price 29. Risk free closeout. For example, $f^+ - f^- = 25bps$ results in NVA=-0.5 circa, 50 bps $\Rightarrow$ NVA = -1
NVA for long call as a function of $f^+ - f^-$, with $f^- = 1\%$, $f^+$ increasing over 1% and $\hat{f}$ increasing accordingly. NVA expressed as a percentage (in bps) of the linearized $\hat{f}$ price. For example, $f^+ - f^- = 25$bps results in NVA=-100bps = -1% circa, replacement closeout relevant (red/blue) for large $f^+ - f^-$. 
Multiple Interest Rate Curves

Derive interest rate dynamics consistently with credit, collateral and funding costs as per the above master valuation equations.
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Derive interest rate dynamics consistently with credit, collateral and funding costs as per the above master valuation equations.

- We use our market-based (no \( r_t \)) master equation to price OIS & find OIS equilibrium rates. Collateral fees will be relevant here, driving forward OIS rates.
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- Use master equation to price also one period swaps based on LIBOR market rates. LIBORs are market given and not modeled from first principles from bonds etc. Forward LIBOR rates obtained by zeroing one period swap and driven both from primitive market LIBOR rates and by collateral fees.
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- In the paper we look at non-perfectly collateralized deals too, where we need to model treasury funding rates.
- See http://ssrn.com/abstract=2244580
Our general theory can be adapted to price under Initial Margins, both under CCPs and SCSA.

The type of equations is slightly different but quantitative problems are quite similar.

Pricing under Initial Margins: SCSA and CCPs

So far all the accounts that need funding have been included within the funding netting set defining $F_t$. 

\[
\phi(t,u) := \int_u^t dv (r v - f v) F_v D(t,v) - \int_u^t dv (f v - h v) H_v D(t,v)
\] (4) 

with $f_{Nt}$ assigned by the Treasury to the initial margin accounts. $f_{N} \neq f$ as initial margins not in funding netting set of the derivative.
Pricing under Initial Margins: SCSA and CCPs

So far all the accounts that need funding have been included within the funding netting set defining $F_t$.

If additional accounts needed, for example segregated initial margins, as with CCP or SCSA, their funding costs must be added.
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So far all the accounts that need funding have been included within the funding netting set defining \( F_t \).

If additional accounts needed, for example segregated initial margins, as with CCP or SCSA, their funding costs must be added.

Initial margins kept into a segregated account, one posted by the investor \( (N_t^I \leq 0) \) and one by the counterparty \( (N_t^C \geq 0) \):

\[
\varphi(t, u) := \int_t^u dv \left( r_v - f_v \right) F_v D(t, v) - \int_t^u dv \left( f_v - h_v \right) H_v D(t, v) \tag{4}
\]

\[
+ \int_t^u dv (f_v^{NC} - r_v) N_v^C + \int_t^u dv (f_v^{NI} - r_v) N_v^I,
\]

with \( f_t^{NC} \) & \( f_t^{NI} \) assigned by the Treasury to the initial margin accounts. \( f^N \neq f \) as initial margins not in funding netting set of the derivative.
Pricing under Initial Margins: SCSA and CCPs

\[ \ldots + \int_t^U dv(f_v^{NC} - r_v)N_v^C + \int_t^U dv(f_v^{NI} - r_v)N_v^I \]

Assume for example \( f > r \). The party that is posting the initial margin has a penalty given by the cost of funding this extra collateral, while the party which is receiving it reports a funding benefit, but only if the contractual rules allow to invest the collateral in low-risk activity, otherwise \( f = r \) and there are no price adjustments.
Pricing under Initial Margins: SCSA and CCPs

\[
\ldots + \int_t^U dv (f^NC_v - r_v) N^C_v + \int_t^U dv (f^NI_v - r_v) N^I_v
\]

Assume for example \( f > r \). The party that is posting the initial margin has a penalty given by the cost of funding this extra collateral, while the party which is receiving it reports a funding benefit, but only if the contractual rules allow to invest the collateral in low-risk activity, otherwise \( f = r \) and there are no price adjustments.

We can describe the default procedure with initial margins and delay by assuming that at 1st default \( \tau \) the surviving party enters a deal with a cash flow \( \vartheta \), at maturity \( \tau + \delta \) (DELAY!).

\( \delta \) 5d (CCP) or 10d (SCSA).
Pricing under Initial Margins: SCSA and CCPs

For a CCP cleared contract priced by the clearing member we have $N^I_{\tau^-} = 0$, whatever the default time, since the clearing member does not post the initial margin.
Pricing under Initial Margins: SCSA and CCPs

For a CCP cleared contract priced by the clearing member we have
\[ N^I_{\tau^-} = 0, \] whatever the default time, since the clearing member does not post the initial margin.

We assume that each margining account accrues continuously at collateral rate \( c_t \).
We may further

- include funding default closeout and also
- define the Initial Margin as a percentile of the mark to market at time \( \tau + \delta \).

This is done explicitly in the paper.

Now a few numerical examples:
Ten-year receiver IRS traded with a CCP.
Prices are calculated from the point of view of the CCP client.
Mid-credit-risk for CCP clearing member, high for CCP client.
Initial margin posted at various confidence levels $q$.

Prices in basis points with a notional of one Euro
Black continuous line: price inclusive of residual CVA and DVA after margining but not funding costs
Dashed black lines represent CVA and the DVA contributions.
Red line is the price inclusive both of credit & funding costs.
Symmetric funding policy. No wrong way correlation overnight/credit.
### CCP Pricing: Tables (see paper for WWR etc)

**Table:** Prices of a ten-year receiver IRS traded with a CCP (or bilaterally) with a mid-risk parameter set for the clearing member (investor) and a high-risk parameter set for the client (counterparty) for initial margin posted at various confidence levels $q$. Prices are calculated from the point of view of the client (counterparty). Symmetric funding policy. WWR correlation $\bar{\rho}$ is zero. Prices in basis points with a notional of one Euro.

<table>
<thead>
<tr>
<th>$q$</th>
<th>Receiver, CCP, $\beta^- = \beta^+ = 1$</th>
<th>Receiver, Bilateral, $\beta^- = \beta^+ = 1$</th>
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<tbody>
<tr>
<td></td>
<td>CVA</td>
<td>DVA</td>
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<tr>
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<td>0.004</td>
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