EEOR E6616: Convex Optimization

Bulletin Description:

Convex sets and functions, and operations preserving convexity. Convex optimization problems.

Convex Duality. Applications of convex optimization problems ranging from signal processing and information theory to revenue management. Convex Optimization in Banach Spaces.

Algorithms for solving constrained convex optimization problems.

Textbook:
Convex Optimization by Boyd and Vandenberghe and detailed course notes.

Pre-requisites: IEOR E6613 Optimization I, EEOR 4605 Convex Optimization for EE

Syllabus

1. Theory of convex sets
   a) Review of basic linear algebra
   b) Hulls: Linear, Affine, Conic, and Convex Hulls
   c) Important examples of convex sets: hyperplanes, half-spaces, norm-balls
   d) Convexity preserving operations: intersection, affine projection, projective maps, etc.
   e) Main geometrical results for convex sets: Caratheodory Theorem, Radon Theorem, Helly's Theorem
   f) Topological properties of convex sets
   g) Separation Theorems: Proper separation, strong separation
   h) Structure of convex sets: recession cone, extreme point, exposed points, etc.
      dual representations of polyhedral convex sets
2. Theory of convex functions
   a) Definition
   b) Convex function = convex set in 1 higher dimension, epigraphs, hypographs, closed convex functions
   c) Convexity preserving operations: non-negative weighted sums, affine transformations, point-wise supremum, partial minimization, conic transformation, etc.
   d) Relation between operations on convex functions and convex sets
   e) Special convex functions: support functions, Minkowski function, Gauge functions, Legendre transforms,
   f) Continuity and differentiability of convex function: sub-differentials, directional derivative, etc.
   g) Quasi-convex and log-concave functions

3. Convex optimization problems
   a) Definition
   b) Optimality conditions
   c) Lagrange duality: relation to Fenchel duality and Legendre transforms
   d) Min-max theorems: min-max for compact sets, min-max for closed functions, Danskin's theorem
   e) Vector convex optimization problems and Pareto optimality

4. Applications of convex and conic programs
   a) Eigenvalue problems
   b) Moment and polynomial positivity problems
   c) Convex relaxations for combinatorial problems
   d) Chapters 6, 7, 8 from Boyd's book.

5. Convex optimization in Banach Spaces
   a) pre-Hilbert and Hilbert spaces
   b) Optimization in Hilbert spaces, Representing Kernel Hilbert Space (RKHS)
   c) Banach spaces: definitions and important examples
   d) Linear functionals on Banach spaces and dual spaces, Hahn-Banach Theorem
e) Norm duality: existence and achievability
f) Lagrange duality in Banach spaces
g) Application: Duality for portfolio selection, Deterministic optimal control

6. First-order algorithms for large scale convex optimization
a) ell-1 and nuclear-norm optimization algorithms
b) Beck-Taboule ISTA/FISTA algorithms
c) Smoothing and Nesterov's fast iterative algorithm for non-smooth convex optimization
d) Augmented Lagrangian algorithms