Where is the Option?
Prepayment Modeling of MBS

April 30, 2007
The Size of the U.S. Mortgage Market

Home mortgages

$10 trillion outstanding
$2.5 trillion originated in 2006

Mortgage-backed Securities (MBS)

$6 trillion total outstanding
Of which $4 trillion are Fannie Mae, Freddie Mac, and Ginnie Mae ("Agency" MBS)

$4.5 trillion Treasury securities outstanding
Conventional U.S. Mortgages

Offered in standardized structures

- Maturity: 30-year or 15-year
- Rate: Fixed, ARM, or hybrid ARM
- Principal payment: amortizing or interest-only
- Borrower can reduce rate by paying points, or avoid closing costs by accepting a higher rate

Prepayable at any time, without penalty

- Refinancing entails transaction cost
What Is an Agency MBS?

Lender/owner of mortgages wants to securitize pool

Say 6% FRMs into a 5 ½% MBS

Agency overlays its guarantee and creates MBS

Principal and specified interest passes through to MBS investors

Residual interest covers servicing cost and guarantee fee
OAS-Based Valuation of Bonds and MBS Since 1986

- Yield Curve and Volatility
- Security Specification
- Option-adjusted Spread (OAS)
- Calibrate Tree (slow)
- Value Security (faster)
- Price
## Bond Valuation Speed Has Improved Dramatically

<table>
<thead>
<tr>
<th></th>
<th>1990</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processor</td>
<td>386 Processor</td>
<td>3.0 GHz Pentium</td>
</tr>
<tr>
<td>Calibrate tree</td>
<td>Up to 4 hours</td>
<td>500 per minute</td>
</tr>
<tr>
<td>Compute fair value recursively</td>
<td>Several seconds</td>
<td>80,000 per minute</td>
</tr>
</tbody>
</table>
MBS Analysis Lags By a Mile

Slow
At most a few 100 of valuations per minute

Imprecise
Monte Carlo simulation, rather than recursion

Prepayment models are derived by statistical analysis, rather than financial principles
Ongoing release of “new and improved” models is a predictable consequence
Typical S-shaped Prepayment Model: Rate Depends on Refinancing Incentive

Figure 14. Refinancing Curves for Different Borrower Types

WAC Weighted-Average Coupon
Source: Salomon Brothers Inc.
MBS Models Can’t Cope with Pertinent “What if” Questions

What if refis were optimal?
(MBS yields would increase by 10 to 15 bps)

What if borrowers gave up the right to refi?
(MBS yields would decline by 45 to 50 bps)

How sensitive are MBS prices to closing costs?

Can prepayments be sensibly modeled without a command of refis?
The right approach to MBS valuation: Understand mortgages; the rest will follow
Analytical Model of Refinancing Decision

**Mortgage: a callable amortizing bond**

Represent call prices as remaining principal plus anticipated refinancing cost

e.g. 1% of remaining principal

**Mortgage rates: OAS to a benchmark curve**

Use a volatility consistent with swaption vols

**Optimum: act only if value received is adequate**

Provides a benchmark for sub-optimal behavior
The Right Decision Tool Is
Generalized Refunding Efficiency

\[ \text{Efficiency}_{\text{gen}} = \frac{PV \text{ Savings}}{\Delta \text{Option Value}} \]
## Should I Refinance?

### CURRENT MORTGAGE
- Years left: 26
- Interest rate (%): 6.000
- Remaining principal ($): 100,000

### NEW MORTGAGE
- Term (in years): 30
- Interest rate (%): 5.600
- Discount points (%): 1.000
- Upfront costs (legal fees, etc.) ($): 1,500
- New principal ($): 102,525

[Refinance Now?](#)  
*patent pending*

## And The Answer Is...

### CASH FLOWS
- Current monthly payment ($): 634
- New monthly payment ($): 588
- Savings per month ($): 46
- Principal remaining after 26 years ($): 25,259

### RECOMMENDATION
- Total savings (in today's $): 1,490
- Loss of option value (in today's $): 2,029

**Kalotay Refi Score**
100% best. Refinancing not recommended below 90%.
Transaction Costs Impede Refinancing (And Increase MBS Value)
Mortgage rates and MBS rates are modeled as a coupled lattice

Each lattice has its own OAS spread relative to benchmark term structure

Mortgage rates determine refis

Using notion of refunding efficiency

MBS cashflows (coupon and principal) discounted on MBS lattice

See “An Option-Theoretic Prepayment Model for Mortgages and Mortgage-Backed Securities” in References
Possible future levels of interest rates ("nodes")

If a mortgagor refinances at this interest rate level ...

... then principal payments pass through to MBS and are discounted on the MBS lattice
A deterministic model of turnover
   Say a specified annual rate
A simple but complete parametrization of interest rate driven refinancings
   A wide spectrum of refinancing behavior
   Pool is divided into several buckets
      • Financial engineers, leapers, and laggards
   For a given behavior, prediction is straightforward
Burnout is a natural consequence of the model
   As “leapers” refinance, pool becomes weighted towards “laggards”
Each mortgagor is assigned an “imputed” coupon representing refinancing behavior.

Refinancing is triggered when a Financial Engineer would refinance a mortgage with the imputed coupon.

A 7% mortgagor whose imputed coupon is 6% is a 1% Laggard.
Laggard Distribution Used In Analysis Below
(Same as in Paper)
Baseline Inputs for Market Testing
September 5, 2006

Environment:
- Reference yield curve
- Swap curve
- Interest rate volatility
- 14%

Mortgagors:
- Turnover rate
- 8%/year
- Mortgagor OAS
- 70 bps
- Refinancing cost (% principal)
- 1%
- Laggard distribution
- Original*

MBS: OAS
- 5 bps

*As in paper
Distribution of Factors in Sample (363 Fannie and Freddie MBS)

Each symbol represents up to 3 observations.
Distribution of “Model – Quote”

Each symbol represents up to 2 observations.
Baseline Inputs Undervalue Recent High-Coupon MBS

*Above-market* WAC reflects weaker credit, which impedes refis

Flat 70 bps OAS overestimates mortgagor’s ability to refi and thus undervalues MBS

The cause suggests a cure

Increase mortgagor OAS to reflect weaker credit

Slows down refis and increases MBS value
“Hockey-stick” Adjustment to Mortgagor OAS

<table>
<thead>
<tr>
<th>WAC (%)</th>
<th>Mortgage OAS (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>70</td>
</tr>
<tr>
<td>5.5</td>
<td>80</td>
</tr>
<tr>
<td>6.0</td>
<td>90</td>
</tr>
<tr>
<td>6.5</td>
<td>100</td>
</tr>
<tr>
<td>7.0</td>
<td>110</td>
</tr>
<tr>
<td>7.5</td>
<td>120</td>
</tr>
<tr>
<td>8.0</td>
<td>130</td>
</tr>
</tbody>
</table>
Model Values of High-Coupon MBS Improve
Resulting in a Much Better Fit

Each symbol represents up to 2 observations.
But Low-Factor Pools Remain Problematic

![Scatter plot showing the relationship between Factor and Difference (% of par). The x-axis represents the Factor ranging from 0.0 to 1.0, and the y-axis represents the Difference (% of par) ranging from -2.5 to 2.5. The data points are represented by red dots, showing a trend where the difference increases as the factor decreases.]
After Removing 9 Low-Factor 6.5% MBS
Summary Statistics of “Model-Quote” (363 MBS)

<table>
<thead>
<tr>
<th>Model Inputs</th>
<th>Mean</th>
<th>Median</th>
<th>Std Dev</th>
<th>1st Quartile</th>
<th>3rd Quartile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.21</td>
<td>-0.07</td>
<td>0.53</td>
<td>-0.40</td>
<td>0.10</td>
</tr>
<tr>
<td>Hockey-stick adjustment</td>
<td>0.07</td>
<td>0.06</td>
<td>0.37</td>
<td>-0.09</td>
<td>0.21</td>
</tr>
<tr>
<td>Hockey-stick, without 9 low-factor MBS</td>
<td>0.05</td>
<td>0.05</td>
<td>0.32</td>
<td>-0.09</td>
<td>0.19</td>
</tr>
</tbody>
</table>

*All units % par*
Mortgagor OAS should reflect borrower’s credit
   For Fannie and Freddie pools, 70 bps over swap curve (strong BBB credit)
   For Ginnie pools, 100bps (weak BBB credit)
Keep other inputs as in example above
   Including laggard distribution and hockey stick adjustment
Estimate MBS OAS from TBA prices
   OAS of whole loans should be roughly 30 bps higher
Refine inputs, depending on precision required
   Low factor pools require special handling
Turnover can be customized
Applications of CLEAN™

End-of-day pricing
850,000 fixed-rate Agency MBS by NYSE/Sector

Electronic trading
Indicative pricing for buy-side platform

Risk management
High-speed stress testing

Real-time portfolio analysis
Why is CLEAN™ a Good Model?

Fully consistent with analysis of swaps and bonds
  Including volatility-dependent option exercise and interpretation of OAS

Expected prepayment behavior is inferred from the market, rather than history
  Just as options trade on implied volatility

User has full control of model
  Including parameterization of refinancing behavior

Quick and accurate calculation of prices and sensitivities
  10,000 recursive valuations per minute
References

“Optimum Bond Calling and Refunding”, W. M. Boyce and Andrew J. Kalotay, *Interfaces* (November 1979)


“Optimum Refinancing: Bringing Professional Discipline to Household Finance”, Andrew Kalotay, Deane Yang, and Frank Fabozzi, working paper

“A Pointer on Points,” Andrew Kalotay and Jinghua Qian, forthcoming in *OR/MS Today*

For Calculators, see [www.kalotay.com/calculators](http://www.kalotay.com/calculators)