Commodities as Financial Assets
Commodities are different, because…

They are produced, consumed, transported, and stored, so…

- Market inventory swings wildly
- Owning a commodity at one place and time is a completely different financial asset from owning it at another. Enforcing arbitrage relationships between them is expensive or impossible
Examples of Traded Commodities

- **Energy**
  - crude oil, gasoline, heating oil, natural gas, electric power, etc

- **Precious Metals**
  - gold, silver, platinum, palladium etc

- **Base Metals**
  - *aluminum, copper, nickel, zinc, etc.*

- **Agricultural**
  - grains, soy beans, coffee, pork bellies, etc

- **Others**
  - pulp, paper, weather, chemicals, etc

Specifications need to be standardized to create trading volume.
The customers of commodity derivatives are industrial producers and consumers, and sometimes governments who depend on the revenue.

Particularly in energy, these customers are particularly risk averse, because of legal sanctions for failure to deliver.
The underlying assets for commodity derivatives are forwards and futures, not spot.

This is a reflection of the statement that the same commodity at a different place or time is a different financial asset.

In addition, hedging with spot is impractical, because spot is much less liquid, and it is impossible to short the spot commodity.
Forwards and Futures on Commodities have special features

Forwards and Futures traded in the market
- Physical forward delivers physical every day for a month, like an average of the spot price
- NYMEX futures, settles on physical forwards
- NYMEX Lookalike forwards, settles on NYMEX future price at expiry
- Publication forwards, e.g. Platt’s, settle on the monthly average of the Platt’s poll of closing spot prices
- Calendar Swap settles on monthly average of closing NYMEX prices
Forwards are referred to in terms of nearbys

The first nearby is simply the forward contract closest to expiry. The second nearby is the second closest, etc.

When a forward contract expires, it is said to “roll off”. The second nearby becomes the first, the third becomes the second, etc.

Most exotic derivatives e.g. barriers and average rates, are written on nearbys, rather than on particular forwards, so that they actually refer to several different forwards.
The Shape of the Forward Curve

There are no curve flattening arbitrages available in commodities.

- If the curve is upward sloping, then buy the earlier forward and sell the later, but one has to take delivery, and store it. Can only make money if price difference is greater than storage costs, defines the “contango limit”.

- If the curve is downward sloping, need to short the spot commodity – impossible.
Behavior of the Forward Curve

- Almost all commodities forward curve have a stable long end, and a violent, whipping short end.
- Long end sits near marginal cost of production
- Short end governed by short term supply and demand
  - If short end is below long end we are in glut == “contango”
  - If short end is above long end we are in shortage == “backwardation”
What is special about commodities forward curves?

- **Backwardation**
What is special about commodities forward curves?

Contango
Bias of the forward curve

- Most trading volume takes place at the long end of the curve – industry buys well in advance.
- Short end of the curve is used to cover unanticipated demand
- Because industry in general, and utilities in particular suffer out of proportion to the trading gain/loss if they fail to deliver, the front of the forward curve is almost always bid up, i.e. backwardated. In financial terms, this translates to extreme risk-aversion
- Investor indices such as GSCI have been invented to allow investors to enter this market, and ride up the forward curve
- Recently, hedge funds have entered the market, generating a large net speculative length
Intermediate points on commodities forward curves tend to have humps at points of anticipated high demand, or supply constraint, and valleys where low demand or high supply are anticipated.

This is mitigated when there is storage capacity covering many more days than the length of the hump or valley.

Natural gas has a large hump in winter, a small one in summer.

Gasoline has a large hump in the “summer driving season.”

Electricity has yearly humps in summer and winter, humps on weekdays, and humps during working hours.
What is special about commodities forward curves?

The Build up to Gulf War I

- 11 Oct 90
- 10 Sep 90
- 10 Aug 90

1 Jan 91
1 Jul 91
1 Feb 92

1 Sep 90
2 Sep 90
3 Sep 90

What is special about commodities forward curves?

**Finer points**

Shape of forward curve affected by available storage and transportation

- Sufficient short-term supply & transport implies short-term contango
  - Aluminum market
Example: Storage and Seasonality in the US Natural Gas Market

- US Natural gas production and consumption average 550bcf/mth.
- We withdraw from November-April ("winter") and store from April-November ("injection season")
- Total NG storage is 3.2 tcf, with a minimum of 500bcf.
- NG forwards are contango in fall leading up to January, and backwardation in spring, with a small summer peak.
- Extreme volatility in the March contract, if it looks like we might not have enough
- But April contract does not reflect this at all!
- If it looks like storage tanks will fill completely before November, can have downward spikes in supply, if it has nowhere to go.
- Transport costs around $0.03/MMBTU, losses around 2%.
What is special about commodities forward curves?

- Regular demand/consumption patterns reflected in the shape of the curve

![Graph showing seasonal variations in energy prices for heating oil and natural gas.](image-url)
**Example: Storage and Seasonality in the US Oil Markets**

- US consumes 22mm bbls/day, and produces about 5mm bbls/day
- Extraction costs range from $2.50/bbl to $12/bbl
- Storage costs $0.15/bbl mth - $0.30/bbl mth, total storage capacity 350mm bbls, with a minimum of 265mm bbls. In addition, there is the US Strategic Petroleum Reserve, but this is held out of the market most of the time.
- Transport costs are about $0.20/ bbl/ kmile.
- Little seasonality in crude oil, but there is seasonality in heating oil, gasoline, etc.
Example: Storage and Seasonality in the US Power Markets

- Power is segmented into separate markets by time of day.
- You can buy either On Peak, or Off Peak, there is a smaller market in individual hours.
- These different times of day have such different properties and pricing that they are regarded as different assets.
- Seasonalities are intra-day, intra-week, and intra-year.
- Power supply is generated by plants with varying efficiencies and start up times, arranged in a generation stack. The most efficient longest startup time plants are at the bottom, and the others are arranged in descending order of efficiency, in a "generation stack".
- Power price jumps with demand as we move up the generation stack.
- It is also possible to transport, if there is spare capacity, but transport between neighboring markets costs 1-5$/MW-hr, out of $35/MW-hr for a typical plant. Also 3% is lost in transmission wires.
What is special about commodities forward curves?

- Monthly, weekly, daily “seasonality” for power
Example: Storage and Seasonality in London Base Metals

- Storage for base metals is cheap, and plentiful
- Transport costs around $0.05/lb - $0.08/lb
- Certain metals have seasonality of demand, but this does not show up in forward curve, possibly because of plentiful storage.
- Aluminum is demanded in summer, by beverage makers
- Lead is demanded in winter, by battery makers
Forward Curves are Frequency-limited by trading from storage operators.

- In order to move the markets for a single futures contract, it takes trading in a volume of size with the same magnitude as the daily usage.
- To stamp out a peak of width T in the forward curve, we need around T days usage in storage (roughly).
- To stamp out a trough of width T in the forward curve, we need around T days usage of storage capacity (roughly).
- Thus, we expect to see details in the forward curve no smaller than the number of days usage in storage, in normal situations.
- When close to the lower limit of storage, we can see finer details in peaks, when close to the upper limit of storage, we can see finer details in troughs.
The volatility surface is made up of options on futures, one option maturity for each futures contract, maturing within a few days (up to a week or two) of the futures maturity. In most markets, the liquid options can range in moneyness from 0.5 to 2, and possibly more.

Because these futures are really different assets, this is not a volatility surface in the usual sense.
Volatilities in commodities markets are almost always backwardated

- Long end moves with long term demand, determined by weather, gdp growth. Very slow, little volatility, 2-10% instantaneous volatility
- Short end whips around with short term supply and demand (200%-300%)
- Reversion occurs over a few weeks.
How volatility term-structure is related to the demand & consumption

*Mean-reverting nature of the market is reflected in the term-structure of volatilities*

Supply/demand imbalances
- excessive “whippiness” of the front end of the forward curve
- high volatility of short-dated options
- Backwardated vol curve

![Graph showing the term-structure of volatilities over time from May 02 to Sep 05.](image-url)
In crises, volatility can become contango
And variance can backwardate!!!

- On occasion there is a supply crunch which affects one month, and not the succeeding one. Implied Volatilities explode for the affected month, but then drop back down for the succeeding month. This can even go to the extent of backwardating the variances. Because one cannot short spot, this cannot be arbitraged.

- In March 2003, this happened in the US Natural Gas markets, because it was a cold winter and we ran out of Natural gas in Texas.
The volatility skew is primarily determined by inventory effects

- Most market participants are industrial, extremely risk averse, hedging exposure.
- Producers want OTM puts, Consumers want OTM calls.
- Market is rarely in balance, and in some cases it is extreme.
- Electricity hedging is only done by producers, vol surface is a diagonal line. ITM puts can be bought at or close to intrinsic value, because dealers are so full of them, they cannot bear further risk.
- Nat Gas hedging is only done by consumers. Skew is very heavy the other way, because the market is all one way.
What is volatility skew, and how is it related to who dominates the market

Scenario 1.

Market dominated by “producers”.

“positive” put skew, “negative” call skew.

![Graph of WTIF03 EXCHANGE Vol Skew](Graph #10)
What is volatility skew, and how is it related to who dominates the market

Scenario 2.

Market dominated by “consumers”. “positive” call skew, “negative” put skew.
What is volatility skew, and how is it related to who dominates the market

Scenario 3.

Market dominated by neither “producers”, nor “consumers”

Skew tend to be fairly symmetric and positive for the calls and puts.
Kurtosis appears immediately, and lasts a long time

- Jumpy behavior visible in observation of futures trading, and in option prices close to expiry.
- Kurtosis is jump-like, in that it appears immediately, does not build up.
- Kurtosis is also Stochastic-vol-like, in that it lasts a long time (more than a year).
- Spikes are present, but do not affect vanilla option value that much.
Non-Black-Scholes Behavior: Spiking

- Comes about when a stored supply is exhausted, or when demand outruns production capacity
- Behavior is difficult to model with Markov models, requires regime-switching, or extreme mean reversion
- Does not really influence value of vanillas, but very important for barriers.
Non-Black-Scholes Behavior: Negative Prices

- Happens in the power markets, because there is no storage, and because it costs a lot of money to shut down and start up certain kinds of plants (nuclear, coal).
- Happens in natural gas markets, but very rarely.
A simple option on the difference between prices in two locations.

Sold as a strip.

Incorporates a loss rate

Can be tricky to model, as correlation is close to 1, yet poorly known, most models are singular at $\rho=1$
Common Commodity Exotics: Load Serving Deals

- Power Utilities would like to hedge not just the power price, but the demand as well, because they cannot refuse to serve.
- The load is also highly correlated with the power price, as well as with weather, and with long term economic growth.
- There is no market in load, so crude models are marked to historic data.
- There are no satisfactory models of load, and almost no work has been done to model it, even though it is critical to many people.
Common Commodity Exotics: Crack Spread Options

- Payoff is the difference between Oil Product (Heating Oil, Fuel Oil) and Crude, minus strike.
- The natural hedge for a refinery.
Common Commod Exotics: Spark Spread Options

- The natural hedge for a gas burning power plant, the payoff is \( \text{Payoff} = \max(P - H \cdot G, 0) \)
- Heat rate \( H \) represents efficiency of the plant, and varies from deal to deal, and from plant to plant.
- For less efficient plants, higher up the generation stack, a strike is sometimes included.
This is an option to hedge out the flexibility that a customer has in buying natural gas.

A customer contracts to buy a certain quantity of natural gas over a series of periods. He has the option to take a certain amount each day, at the floating rate. He must buy at least a minimum amount within the period, or there are penalties. There is rebating in the next period if he buys more than the maximum in a period.

This has a lot of optionality, and is very time-consuming to evaluate, even in a simple model.

This is another interesting problem for academics.
Common Commodity Exotics: Storage Options

- Very similar to Swing options
- A user is rented a storage tank. He has the option each day to buy natgas and inject into the tank, or withdraw and sell natgas from the tank, or do nothing.
- He pays operating fees to inject or withdraw.
- He must return the tank at some level of fill.
- He has a daily injection limit and a daily withdrawal limit.
- This option has a lot of optionality, is difficult and time-consuming to evaluate, even in a simple model
- This is another place where academics can make a real contribution to the business.
Commodities Models: Basic Features

- Spot Price Models
  - Evaluate futures as $F_{tT} = E( S_T | S_t )$, almost always a smooth function (Can’t have discontinuous forward curve!)
  - Almost always have mean reversion
  - Parametrize forward curve with convenience yield $y$
    
    $$F_{tT} = S_t \exp( (r + u - y)(T-t) ), \ u = \text{storage rate}.$$ 
  - Spot models are limited, can’t have negative forward variance in futures. Hard to put in sharply varying forward curves.
  - But Spot models are much more tractable, with fewer factors.
Models of whole curve (i.e. 1 factor for each futures maturity) are capable of encompassing most observed phenomena, but have many more factors, and so are hard to evaluate.

BGM-like Factor models are a kind of compromise.
Commodities Models: Basic Features

- Should have some form of mean reversion
- Should be generalizable to a multi-commodity model, or multi-location model
- A model capturing the vol smile should be calibratable to odd-shaped vol surfaces, distorted by inventory effects.
- A model capturing the vol smile should probably contain jumps.
- A model of storage, transport, or refinery-capacity constrained commodities should include these variables, and their relation to their limits.
Commodities Models: Basic Features

- Market Specific: Natural Gas models may want to use the storage limits, and current value of storage as a parameter, controlling jumpiness, now that there is a forward market in storage numbers.

- Market Specific: Electricity markets should separate different parts of the curve into different assets, hour, day-of-week, season.
Some example models: Gibson-Schwartz Model

- A spot model for electricity, with stochastic convenience yield.
- Cannot accommodate sharply varying forward curves, kurtosis, skew, negative forward variance.
- Does not mean-revert, so variance grows too fast at long times.
Some example models: Schwartz-Smith Model

- A spot model for electricity, modeling spot as a low-vol long term rate, plus a rapidly varying difference, mean reverting to zero.

\[ d\chi = -k\chi\ dt + \sigma_\chi\ dZ_\chi \]
\[ d\zeta = \mu_\zeta\ dt + \sigma_\zeta\ dZ_\zeta \]
\[ \text{Spot} = \zeta + \chi \]

- Cannot accommodate sharply varying forward curves, kurtosis, skew, negative forward variance.

- Has some de-correlation of futures, for time spread options.
Some example models: Gabillon Model

- A spot model for energy, modeling spot as a single factor Gaussian process that mean reverts to a lognormal long term rate
  \[ \frac{dS}{S} = \beta(\ln L - \ln S) \, dt + \sigma_S \, dZ_S \]
  \[ \frac{dL}{L} = \mu_L \, dt + \sigma_L \, dZ_L \]

- Cannot accommodate sharply varying forward curves, kurtosis, skew, negative forward variance.

- Has some de-correlation of futures, for time spread options.
Some example models: Deng Model

- A spot model for electricity and natural gas together, modeling them as 2 mean-reverting models with jumps, and either stochastic vol, or regime switching

\[ dX = K(θ - X) \, dt + M \, dW + ΔZ^1_t + ΔZ^2_t, \]

where \( X \) is a 2-vector containing prices, \( M \) is Cholesky decomp, \( ΔZ^i \) are two \( R^2 \) Poisson processes, one for up and one for down.

- Has kurtosis, skew, spikes!

- Cannot accommodate sharply varying forward curves, negative forward variance.

- Has some de-correlation of futures, for time spread options

- A heavy model to evaluate.
Some example models: Model of Audet, Heiskanen, Keppo and Vehvilainen

- An HJM-like curve model for electricity, in which each forward is a mean-reverting lognormal process.
  \[ \frac{dF_{tT}}{F_{tT}} = \exp(-\alpha(T-t))\sigma(T)dB_{T}(t) \]
  with \( dB_T(t) dB'_T(t) = \exp(-\rho^*|T'-T|) \, dt \)

- Accomodates singular forward curves, and negative forward variance,

- Easy to generalize to multi-commodity

- Forwards nicely de-correlated

- Easy to solve

- Has no skew, kurtosis, jumps, spikes.
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